Math 2584, Spring 2018

Exam 1 will occur during our regularly scheduled class time on Friday, February 14. You are allowed a double-sided, 3 inch by 5 inch card of notes.

(AB 1) Is $y = e^t$ a solution to the differential equation $\frac{d^2y}{dt^2} + \frac{4t}{1-2t}\frac{dy}{dt} - \frac{4}{1-2t}y = 0$?

(AB 2) Is $y = e^{2t}$ a solution to the differential equation $\frac{d^2y}{dt^2} + \frac{4t}{1-2t}\frac{dy}{dt} - \frac{4}{1-2t}y = 0$?

(AB 3) Is $y = e^{3t}$ a solution to the initial value problem $\frac{dy}{dt} = 2y$, $y(0) = 1$?

(AB 4) Is $y = e^{3t}$ a solution to the initial value problem $\frac{dy}{dt} = 3y$, $y(0) = 2$?

(AB 5) Is $y = e^{3t}$ a solution to the initial value problem $\frac{dy}{dt} = 3y$, $y(0) = 1$?

(AB 6) Is $y = e^{3t}$ a solution to the initial value problem $\frac{dy}{dt} = 2y$, $y(0) = 2$?

(AB 7) For each of the following initial-value problems, tell me whether we expect to have an infinite family of solutions, no solutions, or a unique solution. Do not find the solution to the differential equation.

(a) $\frac{dy}{dt} + \arctan(t)\, y = e^t$, $y(3) = 7$.

(b) $\frac{1}{1+t^2}\frac{dy}{dt} - t^5 y = \cos(6t)$, $y(2) = -1$, $y'(2) = 3$.

(c) $\frac{d^2y}{dt^2} - 5\sin(t)\, y = t$, $y(1) = 2$, $y'(1) = 5$, $y''(1) = 0$.

(d) $e^t \frac{d^2y}{dt^2} + 3(t-4)\frac{dy}{dt} + 4t^6 y = 2$, $y(3) = 1$, $y'(3) = -1$.

(e) $\frac{d^2y}{dt^2} + \cos(t)\frac{dy}{dt} + 3\ln(1+t^2)\, y = 0$, $y(2) = 3$.

(f) $\frac{d^3y}{dt^3} - t^7\frac{d^2y}{dt^2} + \frac{dy}{dt} + 5\sin(t)\, y = t^3$, $y(1) = 2$, $y'(1) = 5$, $y''(1) = 0$, $y'''(1) = 3$.

(g) $(1+t^2)\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = t^3$, $y(3) = 9$, $y'(3) = 7$, $y''(3) = 5$.

(h) $\frac{d^3y}{dt^3} - e^t\frac{d^2y}{dt^2} + e^{2t}\frac{dy}{dt} + e^{3t}y = e^{4t}$, $y(-1) = 1$, $y'(-1) = 3$.

(i) $(2+\sin t)\frac{d^3y}{dt^3} + \cos t\frac{d^2y}{dt^2} + \frac{dy}{dt} + 5\sin(t)\, y = t^3$, $y(7) = 2$.

(AB 8) A lake initially has a population of 600 trout. The birth rate of trout is proportional to the number of trout living in the lake. Fishermen are allowed to harvest 30 trout/year from the lake. Write an initial value problem for the fish population. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 9) Five songbirds are blown off course onto an island with no birds on it. The birth rate of songbirds on the island is then proportional to the number of birds living on the island, and the death rate is proportional to the square of the number of birds living on the island. Write an initial value problem for the bird population. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 10) A tank contains hydrogen gas and iodine gas. Initially, there are 50 grams of hydrogen and 3000 grams of iodine. These two gases react to form hydrogen iodide at a rate proportional to the product of the amount of iodine remaining and the amount of hydrogen remaining. The reaction consumes 126.9 grams of iodine for every gram of hydrogen. Write an initial value problem for the amount of hydrogen in the tank. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 11) According to Newton's law of cooling, the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that a cup of coffee is initially at a temperature of 95°C and is placed in a room at a temperature of 25°C. Write an initial value problem for the temperature of the cup of coffee. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 12) Suppose that an isolated town has 300 households. In 1920, two families install telephones in their homes. Write an initial value problem for the number of telephones in the town if the rate at which families buy telephones is jointly proportional to the number of households with telephones and the number of households without telephones. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 13) A large tank initially contains 600 liters of water in which 3 kilograms of salt have been dissolved. Every minute, 2 liters of a salt solution with a concentration of 5 grams per liter flows in, and 3 liters of the well-mixed solution flows out. Write an initial value problem for the amount of salt in the tank. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 14) I want to buy a house. I borrow \$300,000 and can spend \$1600 per month on mortgage payments. My lender charges 4% interest annually, compounded continuously. Suppose that my payments are also made continuously. Write an initial value problem for the amount of money I owe. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 15) A hole in the ground is in the shape of a cone with radius 1 meter and depth 20 cm. Initially, it is empty. Water leaks into the hole at a rate of 1 cm^3 per minute. Water evaporates from the hole at a rate proportional to the area of the exposed surface. Write an initial value problem for the volume of water in the hole. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 16) According to the Stefan-Boltzmann law, a black body in a dark vacuum radiates energy (in the form of light) at a rate proportional to the fourth power of its temperature (in kelvins). Suppose that a planet in space is currently at a temperature of 400 K. Write an initial value problem for the planet's temperature. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

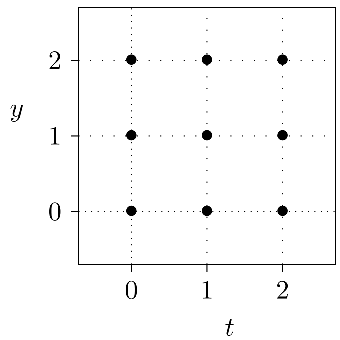
(AB 17) According to the Stefan-Boltzmann law, a black body in a vacuum radiates energy (in the form of light) at a rate proportional to the fourth power of its temperature (in kelvins). If there is some ambient light in the vacuum, then the black body absorbs energy at a rate proportional to the effective temperature of the light. The proportionality constant is the same in both cases; that is, if the object's temperature is equal to the effective temperature of the light, then its temperature will not change.

Suppose that a planet in space is currently at a temperature of 400 K. The effective temperature of its surroundings is 3 K. Write an initial value problem for the planet's temperature. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 18) A ball is thrown upwards with an initial velocity of 10 meters per second. The ball experiences a downwards force due to the Earth's gravity and a drag force proportional to its velocity. Write an initial value problem for the ball's velocity. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

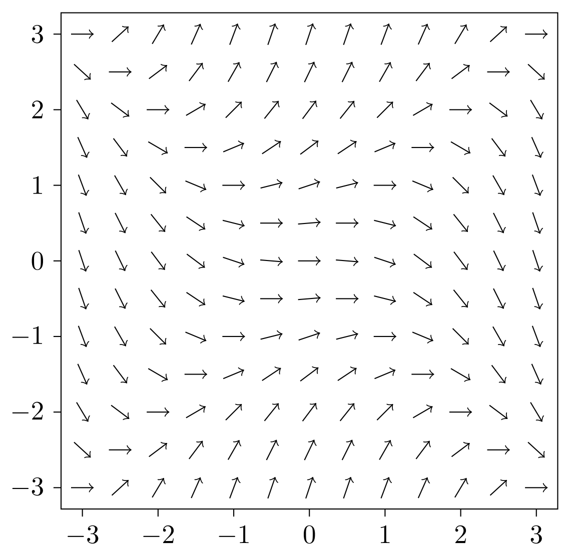
(AB 19) A ball of mass 300 g is thrown upwards with an initial velocity of 20 meters per second. The ball experiences a downwards force due to the Earth's gravity and a drag force proportional to the square of its velocity. Write an initial value problem for the ball's velocity. Specify your independent and dependent variables, any unknown parameters, and your initial condition.

(AB 20) Here is a grid. Draw a small direction field (with nine slanted segments) for the differential equation $\frac{dy}{dt} = t - y$.



(AB 21) Consider the differential equation $\frac{dy}{dx} = (y^2 - x^2)/3$. Here is the direction field for this differential equation. Sketch, approximately, the solution to

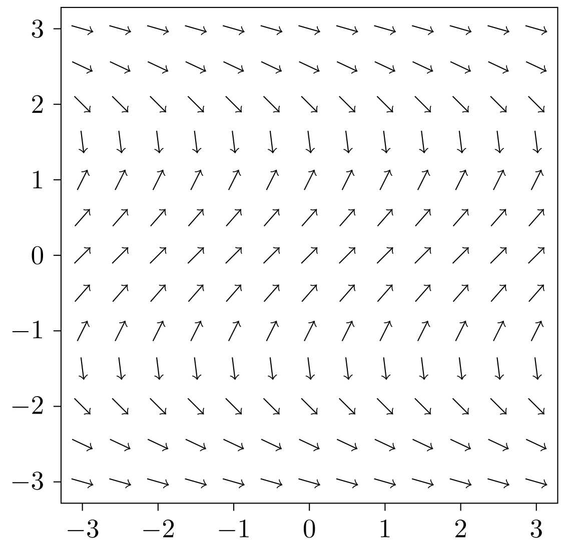
$$\frac{dy}{dx} = (y^2 - x^2)/3, \quad y(-2) = 1.$$



(AB 22) Consider the differential equation $\frac{dy}{dx} = \frac{2}{2-y^2}$. Here is the direction field for this differential equation. Sketch, approximately, the solution to

$$\frac{dy}{dx} = \frac{2}{2-y^2}, \quad y(1) = 0.$$

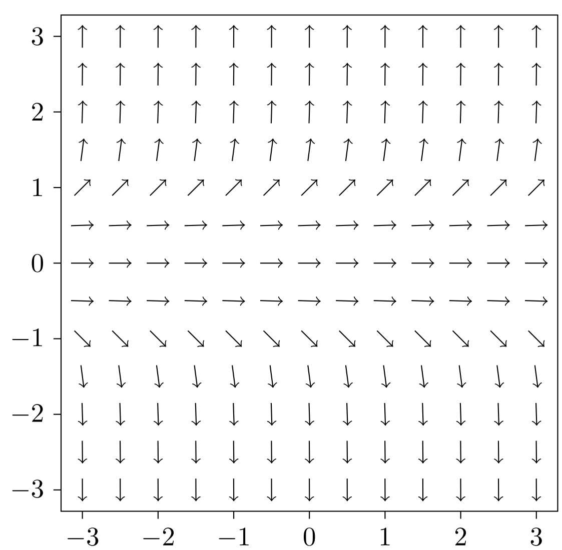
Based on your sketch, what is the (approximate) domain of definition of the solution to this differential equation?



(AB 23) Consider the differential equation $\frac{dy}{dx} = y^5$. Here is the direction field for this differential equation. Sketch, approximately, the solution to

$$\frac{dy}{dx} = y^5, \quad y(1) = -1.$$

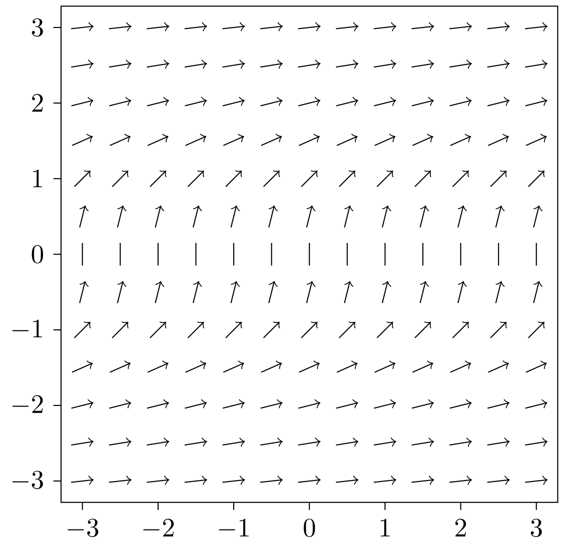
Based on your sketch, what is the (approximate) domain of definition of the solution to this differential equation?



(AB 24) Consider the differential equation $\frac{dy}{dx} = 1/y^2$. Here is the direction field for this differential equation. Sketch, approximately, the solution to

$$\frac{dy}{dx} = \frac{1}{y^2}, \quad y(3) = 2.$$

Based on your sketch, what is the (approximate) domain of definition of the solution to this differential equation?



(AB 25) Consider the autonomous first-order differential equation $\frac{dy}{dx} = (y-2)(y+1)^2$. By hand, sketch some typical solutions.

(AB 26) Find the critical points and draw the phase portrait of the differential equation $\frac{dy}{dx} = \sin y$. Classify each critical point as asymptotically stable, unstable, or semistable.

(AB 27) Find the critical points and draw the phase portrait of the differential equation $\frac{dy}{dx} = y^2(y-2)$. Classify each critical point as asymptotically stable, unstable, or semistable.

(AB 28) I owe my bank a debt of B dollars. The bank assesses an interest rate of 5% per year, compounded continuously. I pay the debt off continuously at a rate of 1600 dollars per month.

(a) Formulate a differential equation for the amount of money I owe.
(b) Find the critical points of this differential equation and classify them as to stability. What is the real-world meaning of the critical points?

(AB 29) A large tank contains 600 liters of water in which salt has been dissolved. Every minute, 2 liters of a salt solution with a concentration of 5 grams per liter flows in, and 2 liters of the well-mixed solution flows out.

(a) Write the differential equation for the amount of salt in the tank.
(b) Find the critical points of this differential equation and classify them as to stability. What is the real-world meaning of the critical points?

(AB 30) A skydiver with a mass of 70 kg falls from an airplane. She deploys a parachute, which produces a drag force of magnitude $2v^2$ newtons, where v is her velocity in meters per second.

(a) Write a differential equation for her velocity. Assume her velocity is always downwards.
(b) Find the (negative) critical points of this differential equation. Be sure to include units.
(c) What is the real-world meaning of these critical points?

(AB 31) A tank contains hydrogen gas and iodine gas. Initially, there are 50 grams of hydrogen and 3000 grams of iodine. These two gases react to form hydrogen iodide at a rate proportional to the product of the amount of iodine remaining and the amount of hydrogen remaining. The reaction consumes 126.9 grams of iodine for every gram of hydrogen.

(*a*) Write a differential equation for the amount of hydrogen left in the tank.
(*b*) Find the critical points of this differential equation. Be sure to include units.
(*c*) What is the real-world meaning of these critical points?

(AB 32) Suppose that the population y of catfish in a certain lake, absent human intervention, satisfies the logistic equation $\frac{dy}{dt} = ry(1 - y/K)$, where r and K are constants and t denotes time.

(*a*) Assuming that $r > 0$ and $K > 0$, find the critical points of this equation and classify them as to stability. What is the long term behavior of the population?
(*b*) Suppose that humans intervene by harvesting fish at a rate proportional to the fish population. Write a new differential equation for the fish population. Find the critical points of your new equation and classify them as to stability. What is the long term behavior of the population?
(*c*) Suppose that humans intervene by harvesting fish at a constant rate. Write a new differential equation for the fish population. Find the critical points of your new equation and classify them as to stability. What is the long term behavior of the population?

(AB 33) For each of the following differential equations, determine whether it is linear, separable, exact, homogeneous, a Bernoulli equation, or of the form $\frac{dy}{dt} = f(At+By+C)$. Then solve the differential equation.

(*a*) $\frac{dy}{dt} = \frac{t+\cos t}{\sin y - y}$
(*b*) $\ln y + y + t + \left(\frac{t}{y} + t\right)\frac{dy}{dt} = 0$
(*c*) $(t^2+1)\frac{dy}{dt} = ty - t^2 - 1$
(*d*) $t^2\frac{dy}{dt} = y^2 + t^2 - ty$
(*e*) $4ty^3 + 6t^3 + (3 + 6t^2y^2 + y^5)\frac{dy}{dt} = 0$
(*f*) $\frac{dy}{dt} = -y\tan 2t - y^3\cos 2t$
(*g*) $(t+y)\frac{dy}{dt} = 5y - 3t$
(*h*) $\frac{dy}{dt} = \frac{1}{3t+2y+7}$.
(*i*) $\frac{dy}{dt} = -y^3\cos(2t)$
(*j*) $4ty\frac{dy}{dt} = 3y^2 - 2t^2$
(*k*) $t\frac{dy}{dt} = 3y - \frac{t^2}{y^5}$
(*l*) $\frac{dy}{dt} = \csc^2(y-t)$.
(*m*) $\frac{dy}{dt} = 8y - y^8$
(*n*) $t\frac{dy}{dt} = -\cos t - 3y$
(*o*) $\frac{dy}{dt} = \cot(y/t) + y/t$

(AB 34) The differential equation $e^x\cos y + (e^x\sin y + 1)\frac{dy}{dx} = 0$ is not exact.

(*a*) Find an integrating factor $\mu(y)$ depending only on y such that, upon multiplying by $\mu(y)$, the equation becomes exact.
(*b*) Solve the differential equation.

(AB 35) The differential equation $3x^2 + 2\sin 2y + x\cos 2y\frac{dy}{dx} = 0$ is not exact.

(*a*) Find an integrating factor $\mu(x)$ depending only on x such that, upon multiplying by $\mu(x)$, the equation becomes exact.
(*b*) Solve the differential equation.

(AB 36) For each of the following differential equations, determine whether it is linear, separable, exact, homogeneous, Bernoulli, or a function of a linear term. Then solve the given initial-value problem.

(a) $\cos(t+y^3)+2t+3y^2\cos(t+y^3)\frac{dy}{dt}=0$, $y(\pi/2)=0$

(b) $2ty\frac{dy}{dt}=4t^2-y^2$, $y(1)=3$.

(c) $t\frac{dy}{dt}=-1-y^2$, $y(1)=1$

(d) $\frac{dy}{dt}=(2y+2t-5)^2$, $y(0)=3$.

(e) $\frac{dy}{dt}=2y-\frac{6}{y^2}$, $y(0)=7$.

(f) $\frac{dy}{dt}=-3y-\sin t\, e^{-3t}$, $y(0)=2$

(AB 37) Solve the initial-value problem $\frac{dy}{dt}=\frac{t-5}{y}$, $y(0)=3$ and determine the range of t-values in which the solution is valid.

(AB 38) Solve the initial-value problem $\frac{dy}{dt}=y^2$, $y(0)=1/4$ and determine the range of t-values in which the solution is valid.

Answer key

(Answer 1) No, $y = e^t$ is not a solution to the differential equation $\frac{d^2y}{dt^2} + \frac{4t}{1-2t}\frac{dy}{dt} - \frac{4}{1-2t}y = 0$.

(Answer 2) Yes, $y = e^{2t}$ is a solution to the differential equation $\frac{d^2y}{dt^2} + \frac{4t}{1-2t}\frac{dy}{dt} - \frac{4}{1-2t}y = 0$.

(Answer 3) No.

(Answer 4) No.

(Answer 5) Yes.

(Answer 6) No.

(Answer 7)

(a) We expect a unique solution.
(b) We do not expect any solutions.
(c) We do not expect any solutions.
(d) We expect a unique solution.
(e) We expect an infinite family of solutions.
(f) We do not expect any solutions.
(g) We expect a unique solution.
(h) We expect an infinite family of solutions.
(i) We expect an infinite family of solutions.

(Answer 8) Independent variable: t = time (in years).
Dependent variable: P = Number of trout in the lake
Initial condition: $P(0) = 600$.
Parameters: α = birth rate (in 1/years).
Differential equation: $\frac{dP}{dt} = \alpha P - 30$.

(Answer 9) Independent variable: t = time (in years).
Dependent variable: P = Number of birds on the island.
Parameters: α = birth rate parameter (in 1/years); β = death rate parameter (in 1/(bird·years)).
Initial condition: $P(0) = 5$.
Differential equation: $\frac{dP}{dt} = \alpha P - \beta P^2$.

(Answer 10) Independent variable: t = time (in minutes).
Dependent variable: H = Amount of hydrogen in the tank (in grams).
Parameters: α = reaction rate parameter (in 1/second·grams).
Initial condition: $H(0) = 50$.
Differential equation: $\frac{dH}{dt} = -\alpha H(126.9H - 3345)$.

(Answer 11) Independent variable: t = time (in seconds).
Dependent variable: T = Temperature of the cup (in degrees Celsius)
Initial condition: $T(0) = 95$.
Differential equation: $\frac{dT}{dt} = -\alpha(T - 25)$, where α is a positive parameter (constant of proportionality) with units of 1/seconds.

(Answer 12) Independent variable: t = time (in years).

Dependent variable: T = Number of telephones installed in the town.

Initial condition: $T(1920) = 2$.

Differential equation: $\frac{dT}{dt} = \alpha T(300 - T)$, where α is a positive parameter (constant of proportionality) with units of $1/\text{year} \cdot \text{telephone}$.

(Answer 13) Independent variable: t = time (in minutes).

Dependent variable: Q = amount of dissolved salt (in kilograms).

Initial condition: $Q(0) = 3$.

Differential equation: $\frac{dQ}{dt} = 0.01 - \frac{3Q}{600-t}$.

(Answer 14) Independent variable: t = time (in years).

Dependent variable: B = balance of my loan (in dollars).

Initial condition: $B(0) = 300{,}000$.

Differential equation: $\frac{dB}{dt} = 0.04B - 12 \cdot 1600$

(Answer 15) Independent variable: t = time (in minutes).

Dependent variables:

h = depth of water in the hole (in centimeters)

V = volume of water in the hole (in cubic centimeters); notice that $V = \frac{1}{3}\pi(5h)^2 h$

Initial condition: $V(0) = 0$.

Differential equation: $\frac{dV}{dt} = 1 - \alpha\pi(5h)^2 = 1 - 25\alpha\pi(3V/25\pi)^{2/3}$, where α is a proportionality constant with units of cm/s.

(Answer 16) Independent variable: t = time (in seconds).

Dependent variable: T = object's temperature (in kelvins)

Parameter: σ = proportionality constant (in $1/(\text{seconds}\cdot\text{kelvin}^3)$)

Initial condition: $T(0) = 400$.

Differential equation: $\frac{dT}{dt} = -\sigma T^4$.

(Answer 17) Independent variable: t = time (in seconds).

Dependent variable: T = object's temperature (in kelvins)

Parameter: σ = proportionality constant (in $1/(\text{seconds}\cdot\text{kelvin}^3)$)

Initial condition: $T(0) = 400$.

Differential equation: $\frac{dT}{dt} = -\sigma T^4 + 81\sigma$.

(Answer 18) Independent variable: t = time (in seconds).

Dependent variable: v = velocity of the ball (in meters/second, where a positive velocity denotes upward motion).

Parameters: α = proportionality constant of the drag force (in newton·seconds/meter)

m = mass of the ball (in kilograms)

Initial condition: $v(0) = 10$.

Differential equation: $\frac{dv}{dt} = -9.8 - (\alpha/m)v$.

(Answer 19) Independent variable: t = time (in seconds).

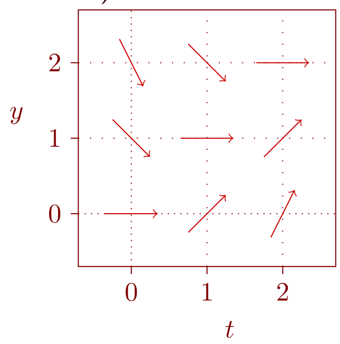
Dependent variable: v = velocity of the ball (in meters/second, where a positive velocity denotes upward motion).

Parameter: α = proportionality constant of the drag force (in newton·seconds/meter).

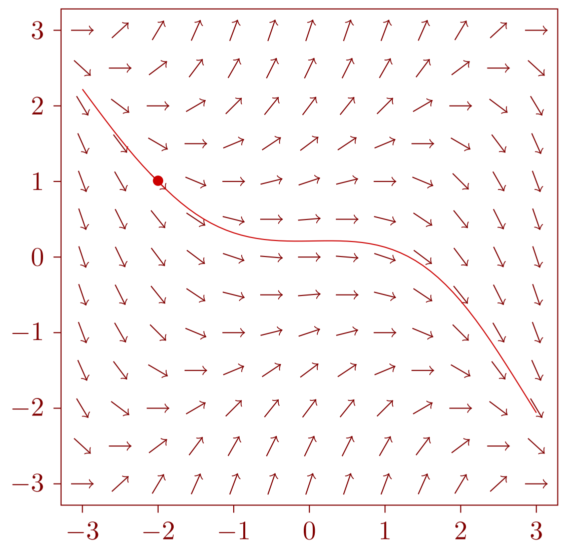
Initial condition: $v(0) = 20$.

Differential equation: $\frac{dv}{dt} = -9.8 - (\alpha/0.3)v|v|$, or $\frac{dv}{dt} = \begin{cases} -9.8 - (\alpha/0.3)v^2, & v \geq 0, \\ -9.8 + (\alpha/0.3)v^2, & v < 0. \end{cases}$

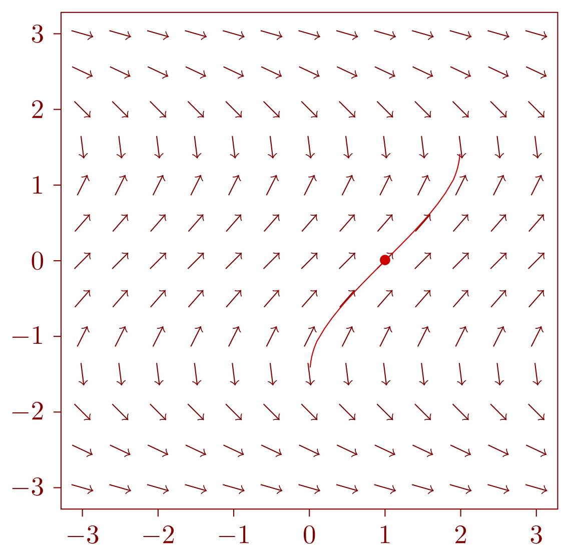
(Answer 20) Here is the direction field for the differential equation $\frac{dy}{dt} = t - y$.



(Answer 21)

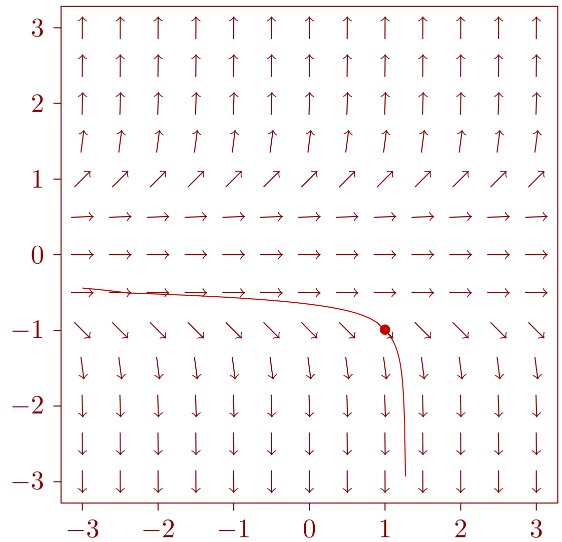


(Answer 22)



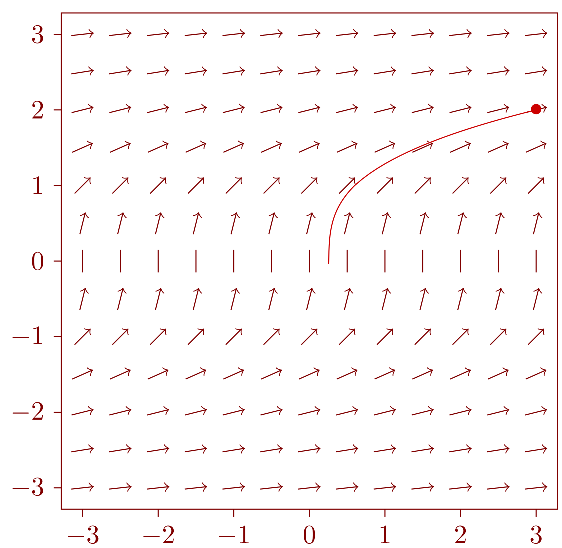
The domain of definition of the solution appears to be $0 < t < 2$.

(Answer 23)



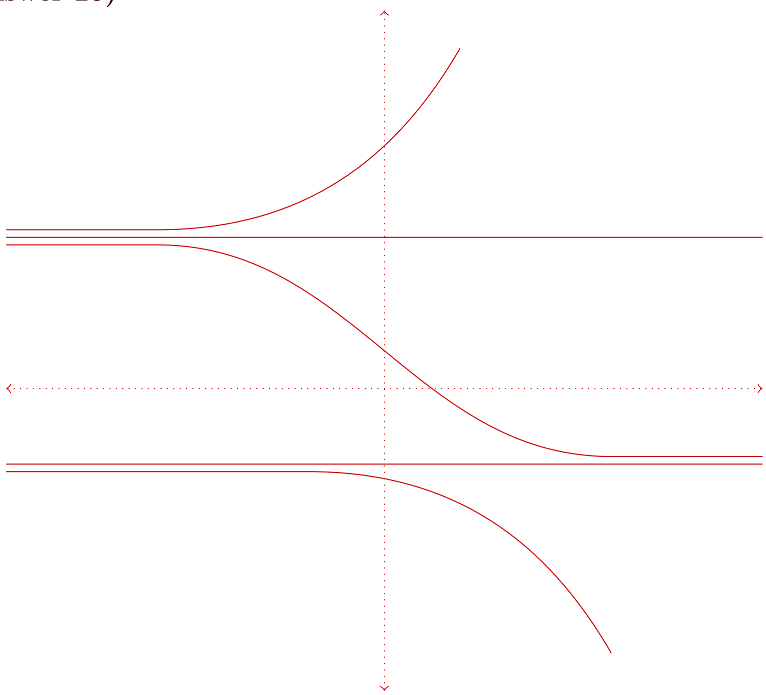
The domain of definition of the solution appears to be approximately $t < 1.3$.

(Answer 24)



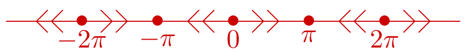
The domain of definition of the solution appears to be $\frac{1}{4} < t$.

(Answer 25)



(Answer 26)

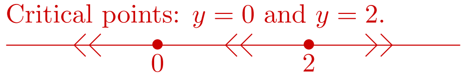
Critical points: $y = k\pi$ for any integer k.



If k is even then $y = k\pi$ is unstable.
If k is odd then $y = k\pi$ is stable.

(Answer 27)

Critical points: $y = 0$ and $y = 2$.



$y = 0$ is semistable. $y = 2$ is unstable.

(Answer 28)

(a) $\frac{dB}{dt} = 0.05B - 19200$, where t denotes time in years.

(b) The critical point is $B = \$384{,}000$. It is unstable. If my initial debt is less than \$384,000, then I will eventually pay it off (have a debt of zero dollars), but if my initial debt is greater than \$384,000, my debt will grow exponentially. The critical point $B = 384{,}000$ corresponds to the balance that will allow me to make interest-only payments on my debt.

(Answer 29)

(a) $\frac{dQ}{dt} = 10 - Q/300$, where Q denotes the amount of salt in grams and t denotes time in minutes.

(b) The critical point is $Q = 3000$. It is stable. No matter the initial amount of salt in the tank, as time passes the amount of salt will approach 3000 g or 3 kg.

(Answer 30)

(a) If $v \leq 0$ then $70\frac{dv}{dt} = -70 * 9.8 + 2v^2$. (If $v > 0$ then $70\frac{dv}{dt} = -70 * 9.8 - 2v^2$.)

(b) $v = -\sqrt{343}$ meters/second.

(c) As $t \to \infty$, her velocity will approach the stable critical point of $v = -\sqrt{343}$ meters/second.

(Answer 31)

(a) $\frac{dH}{dt} = -\alpha H(126.9H - 3345)$, where H is the amount of hydrogen remaining (in grams), t denotes time in minutes, and α is a positive parameter.

(b) $H = 0$ grams and $H = 26.36$ grams.

(c) As $t \to \infty$, the amount of hydrogen in the tank will tend towards 26.36 grams. The $H = 0$ critical point is not physically meaningful, as if $H = 0$ grams then the model predicts -3345 grams of iodine in the tank.

(Answer 32)

(a) The critical points are $y = 0$ (unstable) and $y = K$ (stable). If any positive number of fish are present, then eventually the population will approach a level of $y = K$.

(b) $\frac{dy}{dt} = ry(1 - y/K) - Ey$, where E is a proportionality constant. The critical points are $y = 0$ and $y = K - KE/r$.

If $E < r$, then $y = 0$ is unstable and $y = K - KE/r$ is stable, and the population will approach a level of $y = K - KE/r$.

If $E > r$, then $y = -KE/r + K$ is unstable (but has no physical significance) and 0 is stable. If $E = r$, then $y = 0$ is the only critical point and is semistable. If $E \leq r$, then the population will eventually tend towards extinction.

(c) $\frac{dy}{dt} = ry(1 - y/K) - h$, where h is the harvesting rate. If $h < Kr/4$, then the critical points are $y = K/2 - \sqrt{(K/2)^2 - Kh/r}$ (unstable) and $y = K/2 + \sqrt{(K/2)^2 - Kh/r}$ (stable). Notice that both critical points are positive (i.e., correspond to the physically meaningful case of at least zero fish in the lake.) If $h = Kr/4$, then there is one critical point at $y = K/2$; it is semistable. Finally, if $h > Kr/4$, then there are no critical points and the fish population will decrease until it goes extinct.

(Answer 33)

(a) $\frac{dy}{dt} = \frac{t+\cos t}{\sin y - y}$ is separable and has solution $\frac{1}{2}y^2 + \cos y = -\frac{1}{2}t^2 - \sin t + C$.

(b) $\ln y + y + t + \left(\frac{t}{y} + t\right)\frac{dy}{dt} = 0$ is exact and has solution $t\ln y + ty + \frac{1}{2}t^2 = C$.

(c) $(t^2+1)\frac{dy}{dt} = ty - t^2 - 1$ is linear and has solution $y = -\sqrt{t^2+1}\ln(t + \sqrt{t^2+1}) + C\sqrt{t^2+1}$.

(d) $t^2\frac{dy}{dt} = y^2 + t^2 - ty$ is homogeneous and has solution $y = \frac{t}{C - \ln|t|} + t$.

(e) $4ty^3 + 6t^3 + (3 + 6t^2y^2 + y^5)\frac{dy}{dt} = 0$ is exact and has solution $2t^2y^3 + \frac{3}{2}t^4 + 3y + \frac{1}{6}y^6 = C$.

(f) $\frac{dy}{dt} = -y\tan 2t - y^3\cos 2t$ is Bernoulli. Let $v = y^{-2}$. Then $\frac{dv}{dt} = 2v\tan 2t + 2\cos 2t$, so $v = \frac{1}{2}\sin(2t) + t\sec(2t) + C\sec(2t)$ and $y = \frac{1}{\sqrt{\frac{1}{2}\sin(2t) + t\sec(2t) + C\sec(2t)}}$.

(g) $(t+y)\frac{dy}{dt} = 5y - 3t$ is homogeneous and has solution $(y - 3t)^2 = C(y - t)$.

(h) If $\frac{dy}{dt} = \frac{1}{3t+2y+7}$, then $\frac{3t+2y+7}{3} - \frac{2}{9}\ln|3t + 2y + 7 + 2/3| = \ln|t| + C$.

(i) $\frac{dy}{dt} = -y^3\cos(2t)$ is separable and has solution $y = \pm\frac{1}{\sqrt{\sin(2t)+C}}$.

(j) $4ty\frac{dy}{dt} = 3y^2 - 2t^2$ is homogeneous (and also Bernoulli) and has solution $2\ln(y^2/t^2 + 2) = -\ln|t| + C$.

(k) $t\frac{dy}{dt} = 3y - \frac{t^2}{y^5}$ is a Bernoulli equation. Let $v = y^6$. Then $t\frac{dv}{dt} = 18v - 6t^2$, so $v = Ct^{18} + \frac{3}{8}t^2$ and $y = \sqrt[6]{Ct^{18} + \frac{3}{8}t^2}$.

(l) If $\frac{dy}{dt} = \csc^2(y - t)$, then $\tan(y - t) - y = C$.

(m) $\frac{dy}{dt} = 8y - y^8$ is Bernoulli (and also separable, but separating variables results in an impossible integral). Make the substitution $v = y^{-7}$. Then $\frac{dv}{dt} = -56v + 7$, so $v = Ce^{-56t} + \frac{1}{8}$ and $y = \frac{1}{\sqrt[7]{Ce^{-56t} + 1/8}}$.

(n) $t\frac{dy}{dt} = -\cos t - 3y$ is linear and has solution $y = \frac{1}{t}\sin t + \frac{2}{t^2}\cos t - \frac{2}{t^3}\sin t + \frac{C}{t^3}$.

(o) $\frac{dy}{dt} = \cot(y/t) + y/t$ is homogeneous and has solution $\sec(y/t) = Ct$.

(Answer 34)

(a) $\mu(y) = \sec^2 y$.

(b) $e^x\sec y + \tan y = C$.

(Answer 35)

(a) $\mu(x) = x^3$.

(b) $\frac{1}{2}x^4 \sin 2y + \frac{1}{2}x^6 = C$.

(Answer 36)

(a) If $\cos(t + y^3) + 2t + 3y^2 \cos(t + y^3)\frac{dy}{dt} = 0$, $y(\pi/2) = 0$, then $\sin(t + y^3) + t^2 = 1 + \pi^2/4$.

(b) If $2ty\frac{dy}{dt} = 4t^2 - y^2$, $y(1) = 3$, then $y = \sqrt{\frac{4}{3}t^2 + \frac{23}{3t}}$.

(c) If $t\frac{dy}{dt} = -1 - y^2$, $y(1) = 1$, then $y = \tan(\pi/4 - \ln t)$.

(d) If $\frac{dy}{dt} = (2y + 2t - 5)^2$, $y(0) = 3$, then $y = \frac{1}{2}\tan(2t + \pi/4) + \frac{5}{2} - t$.

(e) If $\frac{dy}{dt} = 2y - \frac{6}{y^2}$, $y(0) = 7$, then $y = \sqrt[3]{340e^{6t} + 3}$.

(f) If $\frac{dy}{dt} = -3y - \sin t\, e^{-3t}$, $y(0) = 2$, then $y = e^{-3t}\cos t + e^{-3t}$.

(Answer 37) $y = \sqrt{(t-5)^2 - 16} = \sqrt{(t-1)(t-9)} = \sqrt{t^2 - 10t + 9}$. The solution is valid for all $t < 1$.

(Answer 38) $y = \frac{1}{4-t}$. The solution is valid for all $t < 4$.