Math 3083, Spring 2019

(Problem 1) Use back substitution to solve the system

$$\begin{cases} 3x_1 - 2x_2 + x_3 = 5, \\ - 5x_2 - 2x_3 = 1, \\ 3x_3 = 7. \end{cases}$$

(Problem 2) In each of the following systems, interpret each equation as a line in the plane. For each system, graph the lines and determine geometrically the number of solutions.

$(a)\ \begin{cases} 3x_1 - 2x_2 = 6, \\ -6x_1 + 4x_2 = -12. \end{cases}$ $(b)\ \begin{cases} 3x_1 - 2x_2 = 6, \\ -4x_1 + 2x_2 = 8. \end{cases}$ $(c)\ \begin{cases} 4x_1 - 2x_2 = 8, \\ -2x_1 + x_2 = 6. \end{cases}$

(Problem 3) Solve each of the following systems by writing the augmented matrix and finding a row equivalent matrix in strict upper triangular or reduced row echelon form. Indicate whether the system is consistent. If it is, find the solution set.

$(a)\ \begin{cases} 4x_1 + x_2 - 5x_3 = -9, \\ 2x_1 - 5x_2 + 3x_3 = 1, \\ 3x_1 + 2x_2 - 2x_3 = 1. \end{cases}$ $(b)\ \begin{cases} 2x_1 + 6x_2 - 3x_3 = 2, \\ 3x_1 + 9x_2 + x_3 = 14, \\ x_1 + 3x_2 - 2x_3 = 0. \end{cases}$ $(c)\ \begin{cases} 2x_1 - x_2 - 4x_3 = 2, \\ 4x_1 + 3x_2 + 2x_3 = 25, \\ 3x_1 + 4x_2 + 5x_3 = 10. \end{cases}$

(Problem 4) Which of the following matrices are in reduced row echelon form?

$(a)\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $(k)\ \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $(u)\ \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ $(ee)\ \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$

$(b)\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $(l)\ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $(v)\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ $(ff)\ \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

$(c)\ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ $(m)\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $(w)\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $(gg)\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$(d)\ \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix}$ $(n)\ \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ $(x)\ \begin{pmatrix} 1 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $(hh)\ \begin{pmatrix} 2 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

$(e)\ \begin{pmatrix} 3 & 7 \\ 0 & 0 \end{pmatrix}$ $(o)\ \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 4 \end{pmatrix}$ $(y)\ \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 6 \end{pmatrix}$ $(ii)\ \begin{pmatrix} 1 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

$(f)\ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $(p)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(z)\ \begin{pmatrix} 1 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ $(jj)\ \begin{pmatrix} 1 & 3 & 7 \\ 0 & 0 & 0 \end{pmatrix}$

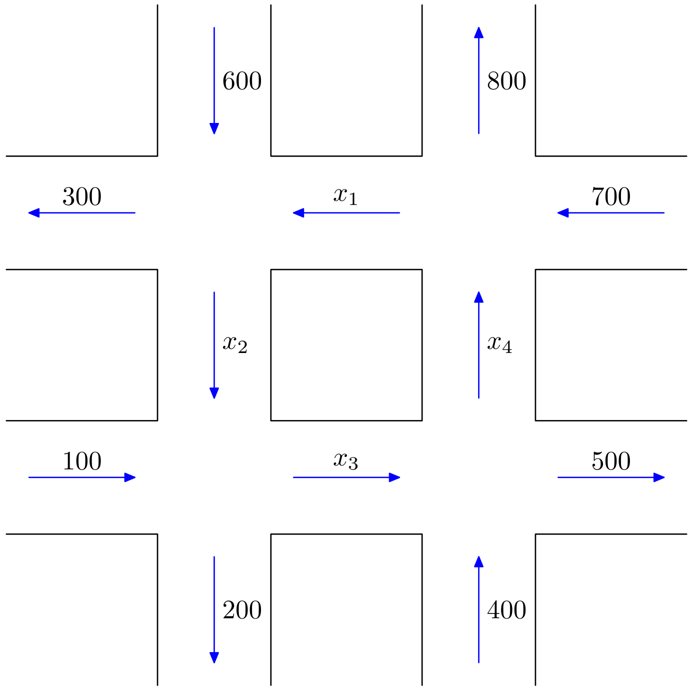
$(g)\ \begin{pmatrix} 0 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(q)\ \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{pmatrix}$ $(aa)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $(kk)\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$(h)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(r)\ \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(bb)\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $(ll)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

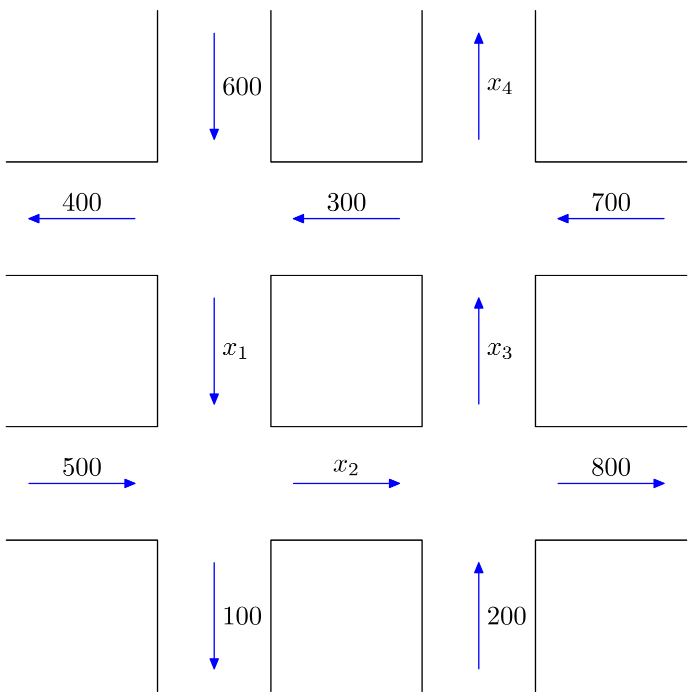
$(i)\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{pmatrix}$ $(s)\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 6 \end{pmatrix}$ $(cc)\ \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $(mm)\ \begin{pmatrix} 0 & 3 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$(j)\ \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 5 \end{pmatrix}$ $(t)\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $(dd)\ \begin{pmatrix} 1 & 3 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(nn)\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

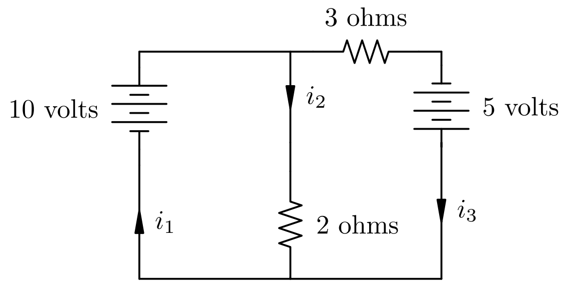
(Problem 5) Determine all possible values of x_1, x_2, x_3, and x_4 in the following traffic flow diagram.



(Problem 6) Determine all possible values of x_1, x_2, x_3, and x_4 in the following traffic flow diagram.



(Problem 7) Determine the values of the currents i_1, i_2, and i_3 in the following circuit diagram.



(Problem 8) Carbon monoxide reacts with hydrogen to produce octane and water. The chemical equation for this reaction is of the form $x_1\mathrm{CO} + x_2\mathrm{H}_2 \to x_3\mathrm{C}_8\mathrm{H}_{18} + x_4\mathrm{H}_2\mathrm{O}$. Determine (nonzero integer) values of x_1, x_2, x_3 and x_4 to balance the equation.

(Problem 9) Find the following products or state that they are not meaningful.

(a) $\begin{pmatrix} 6 & 2 & 1 \\ 3 & 4 & 8 \\ 0 & 7 & 5 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 3 \end{pmatrix}$. (c) $\begin{pmatrix} 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 6 & 2 & 1 \\ 3 & 4 & 8 \\ 0 & 7 & 5 \end{pmatrix}$. (d) $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 & 4 & 1 \end{pmatrix}$.

(Problem 10) Write the system $\begin{cases} 4x_1 + x_2 - 5x_3 = -9, \\ 2x_1 - 5x_2 + 3x_3 = 1, \\ 3x_1 + 2x_2 - 2x_3 = 1. \end{cases}$ as a matrix equation.

(Problem 11) Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix}$. Write $\vec{b} = \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix}$ as a linear combination of the columns of A. Then solve $A\vec{x} = \vec{b}$.

(Problem 12) Find a nonzero matrix A such that $A^2 = O$.

(Problem 13) Find a nonzero 2×2 matrix A such that $A \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(Problem 14) Compute the inverses of the following matrices.

(a) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 4 \\ 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 4 \\ 0 & 2 \end{pmatrix}$

(Problem 15) Let $A = \begin{pmatrix} 2/3 & 2/3 \\ 1/3 & 1/3 \end{pmatrix}$. Find A^2.

(Problem 16) Find an elementary matrix E such that $\begin{pmatrix} 5 & 7 & 9 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 17) Find an elementary matrix E such that $\begin{pmatrix} 1 & 2 & 3 \\ -10 & -13 & -10 \\ 7 & 9 & 8 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 18) Find an elementary matrix E such that $\begin{pmatrix} 4 & 8 & 12 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 19) Find an elementary matrix E such that $\begin{pmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 9 & 8 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 20) Find an elementary matrix E such that $\begin{pmatrix} 1 & 2 & 3 \\ -8 & -10 & -12 \\ 7 & 9 & 8 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 21) Find an elementary matrix E such that $\begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 21 & 27 & 24 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 22) Find an elementary matrix E such that $\begin{pmatrix} 7 & 9 & 8 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 23) Find an elementary matrix E such that $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 10 & 15 & 17 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 24) Find an elementary matrix E such that $\begin{pmatrix} 1 & 2 & 3 \\ 7 & 9 & 8 \\ 4 & 5 & 6 \end{pmatrix} = E \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{pmatrix}$.

(Problem 25)

(a) Find three elementary matrices E_3, E_2 and E_1 such that $E_3E_2E_1 \begin{pmatrix} 2 & 5 & 4 \\ 6 & 2 & 1 \\ -4 & 3 & 5 \end{pmatrix}$ is upper triangular.

(b) Find a LU factorization of $M = \begin{pmatrix} 2 & 5 & 4 \\ 6 & 2 & 1 \\ -4 & 3 & 5 \end{pmatrix}$. That is, find an upper triangular matrix U and a lower triangular matrix L, where the diagonal entries of L are all 1s, such that $LU = M$.

(Problem 26) Compute the inverses of the following matrices.

(a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(Problem 27) Compute the inverses of the following matrices.

(a) $\begin{pmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 0 & 3 \\ 1 & 3 & -2 \\ 2 & 5 & -3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 8 & 4 \\ 4 & 0 & 3 \\ 1 & 5 & 2 \end{pmatrix}$

(Problem 28) Use your answers to Problem 27 to find the inverses of the following matrices.

(a) $\begin{pmatrix} 1 & 4 & 7 \\ 3 & 5 & 8 \\ 2 & 6 & 9 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 5 \\ 3 & -2 & -3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 4 & 1 \\ 8 & 0 & 5 \\ 4 & 3 & 2 \end{pmatrix}$

(Problem 29) Use your answers to Problem 27 to solve the following equations.

(a) $\begin{pmatrix} 2 & 8 & 4 \\ 4 & 0 & 3 \\ 1 & 5 & 2 \end{pmatrix} \vec{x} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ $\qquad$ (b) $\begin{pmatrix} 4 & 0 & 3 \\ 1 & 3 & -2 \\ 2 & 5 & -3 \end{pmatrix} X = \begin{pmatrix} 7 & 2 \\ 3 & 5 \\ 2 & 1 \end{pmatrix}$

(Problem 30) Let $A = \begin{pmatrix} 0 & 2 & -1 & 2 & 1 \\ 1 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 1 & 2 & 0 & -3 & 0 \\ -1 & 1 & 0 & 3 & 0 \end{pmatrix}$.

(a) Find $A \begin{pmatrix} 3 \\ 0 \\ 3 \\ 1 \\ 1 \end{pmatrix}$.

(b) Is A singular? How do you know?

(Problem 31) Let $A = \begin{pmatrix} 1 & 0 & -1 & 2 & 1 \\ 1 & 0 & -2 & 0 & 2 \\ 1 & 0 & 2 & -1 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ -1 & 0 & -2 & 1 & 0 \end{pmatrix}$.

(a) Find $A \begin{pmatrix} -3 \\ 5 \\ 8 \\ 4 \\ 10 \end{pmatrix}$.

(b) Find $A \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$.

(c) Is A singular? How do you know?

(Problem 32) You are given that $A = \begin{pmatrix} 3 & 5 & 7 & 2 & 1 \\ 1 & 5 & -2 & 3 & 2 \\ 1 & 7 & 2 & -1 & 2 \\ 2 & 1 & 6 & 3 & 0 \\ -1 & 9 & -2 & 4 & 3 \end{pmatrix}$ is invertible. Is there a solution to $\begin{pmatrix} 3 & 5 & 7 & 2 & 1 \\ 1 & 5 & -2 & 3 & 2 \\ 1 & 7 & 2 & -1 & 2 \\ 2 & 1 & 6 & 3 & 0 \\ -1 & 9 & -2 & 4 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$? (You don't have to find $\vec{x}$.)

(Problem 33) You are given that there are no solutions to $\begin{pmatrix} 3 & 5 & 7 & 2 & 1 \\ 3 & 5 & -2 & 3 & 2 \\ 1 & 14 & 4 & -2 & 4 \\ 2 & 1 & 6 & 3 & 0 \\ -1 & 6 & -2 & 4 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$. Is $A = \begin{pmatrix} 3 & 5 & 7 & 2 & 1 \\ 3 & 5 & -2 & 3 & 2 \\ 1 & 14 & 4 & -2 & 4 \\ 2 & 1 & 6 & 3 & 0 \\ -1 & 6 & -2 & 4 & 3 \end{pmatrix}$ invertible? How do you know?

Answer key

(Answer 1) $x_3 = 7/3$, $x_2 = -17/15$, $x_1 = 2/15$.

(Answer 2)

(a) There are infinitely many solutions.
(b) There is exactly one solution.
(c) There are no solutions.

(Answer 3)

(a) The augmented matrix is $\left(\begin{array}{ccc|c} 4 & 1 & -5 & -9 \\ 2 & -5 & 3 & 1 \\ 3 & 2 & -2 & 1 \end{array}\right)$.

The system is consistent. The only solution is $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.

(b) The augmented matrix is $\left(\begin{array}{ccc|c} 2 & 6 & -3 & 2 \\ 3 & 9 & 1 & 14 \\ 1 & 3 & -2 & 0 \end{array}\right)$. It is row equivalent to $\left(\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$.

The system is consistent. The solution set is $\{(4 - 3\alpha, \alpha, 2)\}$ or $\{(x_1, x_2, 2) : x_1 = 4 - 3x_2\}$.

(c) The augmented matrix is $\left(\begin{array}{ccc|c} 2 & -1 & -4 & 2 \\ 4 & 3 & 2 & 25 \\ 3 & 4 & 5 & 10 \end{array}\right)$. It is row equivalent to $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right)$.

The system is inconsistent.

(Answer 4)

(*a*) Yes.
(*b*) Yes.
(*c*) Yes.
(*d*) No.
(*e*) No.
(*f*) No.
(*g*) No.
(*h*) Yes.
(*i*) No.
(*j*) No.
(*k*) No.
(*l*) Yes.
(*m*) Yes.
(*n*) No.
(*o*) No.
(*p*) Yes.
(*q*) Yes.
(*r*) Yes.
(*s*) No.
(*t*) Yes.
(*u*) Yes.
(*v*) No.
(*w*) Yes.
(*x*) Yes.
(*y*) Yes.
(*z*) No.
(*aa*) No.
(*bb*) Yes.
(*cc*) Yes.
(*dd*) Yes.
(*ee*) No.
(*ff*) No.
(*gg*) Yes.
(*hh*) No.
(*ii*) No.
(*jj*) Yes.
(*kk*) Yes.
(*ll*) No.
(*mm*) No.
(*nn*) No.

(Answer 5) $\{(\alpha-100, \alpha+200, \alpha+100, \alpha)\}$ or $\{(x_1, x_2, x_3, x_4) : x_1 = x_4-100, x_2 = x_4+200, x_3 = x_4+100\}$.

(Answer 6) $x_1 = 500$, $x_2 = 900$, $x_3 = 300$, $x_4 = 700$.

(Answer 7) $i_1 = 10$ amperes, $i_2 = 5$ amperes, $i_3 = 5$ amperes.

(Answer 8) $x_1 = 8$, $x_2 = 17$, $x_3 = 1$, $x_4 = 8$, or any multiple of those numbers.

(Answer 9)

(a) $\begin{pmatrix} 6 & 2 & 1 \\ 3 & 4 & 8 \\ 0 & 7 & 5 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -12 & 29 \\ -3 & 29 \end{pmatrix}$.

(b) The product is not meaningful.

(c) $\begin{pmatrix} 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \end{pmatrix}$.

(d) $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 6 & 8 & 2 \\ 9 & 12 & 3 \end{pmatrix}$

(Answer 10) $\begin{pmatrix} 4 & 1 & -5 \\ 2 & -5 & 3 \\ 3 & 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \\ 1 \end{pmatrix}$.

(Answer 11) $\vec{b} = 2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$. The solution to $A\vec{x} = \vec{b}$ is $\vec{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(Answer 12) There are many possible answers, including $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $A = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix}$.

(Answer 13) There are many possible answers, including $A = \begin{pmatrix} 2 & -3 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ -2 & 3 \end{pmatrix}$, $\begin{pmatrix} 4 & -6 \\ -6 & 9 \end{pmatrix}$.

(Answer 14)

(a) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$.

(b) $\begin{pmatrix} 0 & 4 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -1/8 & 1/2 \\ 1/4 & 0 \end{pmatrix}$.

(c) $\begin{pmatrix} 3 & 4 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/3 & -2/3 \\ 0 & 1/2 \end{pmatrix}$.

(Answer 15) $A^2 = A$.

(Answer 16) $E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(Answer 17) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$.

(Answer 18) $E = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(Answer 19) $E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(Answer 20) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(Answer 21) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

(Answer 22) $E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$.

(Answer 23) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$.

(Answer 24) $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

(Answer 25)

(a) One possible answer is $E_1 = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$, $E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

Another possible answer is $E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

(b) $U = \begin{pmatrix} 2 & 5 & 4 \\ 0 & -13 & -11 \\ 0 & 0 & 2 \end{pmatrix}$, $L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & -1 & 1 \end{pmatrix}$.

(Answer 26)

(a) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

(Answer 27)

(a) $\begin{pmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} -0.333333333333 & -1.22222222222 & 0.888888888889 \\ 0.666666666667 & -0.555555555556 & 0.222222222222 \\ -0.333333333333 & 1.44444444444 & -0.777777777778 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 0 & 3 \\ 1 & 3 & -2 \\ 2 & 5 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 15 & -9 \\ -1 & -18 & 11 \\ -1 & -20 & 12 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 8 & 4 \\ 4 & 0 & 3 \\ 1 & 5 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -1.5 & 0.4 & 2.4 \\ -0.5 & 0.0 & 1.0 \\ 2.0 & -0.2 & -3.2 \end{pmatrix}$

(Answer 28)

(a) $\begin{pmatrix} 1 & 4 & 7 \\ 3 & 5 & 8 \\ 2 & 6 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} -0.333333333333 & 0.666666666667 & -0.333333333333 \\ -1.22222222222 & -0.555555555556 & 1.44444444444 \\ 0.888888888889 & 0.222222222222 & -0.777777777778 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 5 \\ 3 & -2 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & -1 \\ 15 & -18 & -20 \\ -9 & 11 & 12 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 4 & 1 \\ 8 & 0 & 5 \\ 4 & 3 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} -1.5 & -0.5 & 2.0 \\ 0.4 & 0.0 & -0.2 \\ 2.4 & 1.0 & -3.2 \end{pmatrix}$

(Answer 29)

(a) $\vec{x} = \begin{pmatrix} 8.7 \\ 3.5 \\ -10.4 \end{pmatrix}$.

(b) $X = \begin{pmatrix} 34 & 68 \\ -39 & -81 \\ -43 & -90 \end{pmatrix}$.

(Answer 30)

(a) $\begin{pmatrix} 0 & 2 & -1 & 2 & 1 \\ 1 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 1 & 2 & 0 & -3 & 0 \\ -1 & 1 & 0 & 3 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

(b) Yes; if $A\vec{x} = \vec{0}$ has a nonzero solution $\vec{x}$ then A is singular.

(Answer 31)

(a) $\begin{pmatrix} 1 & 0 & -1 & 2 & 1 \\ 1 & 0 & -2 & 0 & 2 \\ 1 & 0 & 2 & -1 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ -1 & 0 & -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 5 \\ 8 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 9 \\ 19 \\ -9 \end{pmatrix}$.

(b) $\begin{pmatrix} 1 & 0 & -1 & 2 & 1 \\ 1 & 0 & -2 & 0 & 2 \\ 1 & 0 & 2 & -1 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ -1 & 0 & -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ 9 \\ 19 \\ -9 \end{pmatrix}$.

(c) Yes; if there is a vector $\vec{b}$ such that $A\vec{x} = \vec{b}$ has more than one solution, then A is singular.

(Answer 32) Yes; $\vec{x} = \begin{pmatrix} 3 & 5 & 7 & 2 & 1 \\ 1 & 5 & -2 & 3 & 2 \\ 1 & 7 & 2 & -1 & 2 \\ 2 & 1 & 6 & 3 & 0 \\ -1 & 9 & -2 & 4 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$.

(Answer 33) No; if A were invertible, then there would be a solution $A^{-1}\vec{b}$ to $A\vec{x} = \vec{b}$ for every $\vec{b}$ of appropriate length.