Math 2584, Spring 2018

You are allowed a double-sided, 3 inch by 5 inch card of notes.

You are responsible for all of the formulas you will need, except for the following Laplace transforms, which will be written on the last page of the exam.

$$
\begin{aligned}
\mathcal{L}\{f(t)\} &= \int_0^\infty e^{-st} f(t)\,dt \\
\mathcal{L}\{1\} &= \frac{1}{s}, && s > 0, \\
\mathcal{L}\{t\} &= \frac{1}{s^2}, && s > 0, \\
\mathcal{L}\{t^n\} &= \frac{n!}{s^{n+1}}, && s > 0, && n \geq 0 \\
\mathcal{L}\{e^{at}\} &= \frac{1}{s-a}, && s > a \\
\mathcal{L}\{\cos(kt)\} &= \frac{s}{s^2+k^2}, && s > 0 \\
\mathcal{L}\{\sin(kt)\} &= \frac{k}{s^2+k^2}, && s > 0 \\
\mathcal{L}\{\cosh(kt)\} &= \frac{s}{s^2-k^2}, && s > |k| \\
\mathcal{L}\{\sinh(kt)\} &= \frac{k}{s^2-k^2}, && s > |k| \\
\text{If } \mathcal{L}\{f(t)\} &= F(s) \text{ then } \mathcal{L}\{e^{at} f(t)\} = F(s-a), \\
\mathcal{L}^{-1}\{F(s)\} &= e^{at}\,\mathcal{L}^{-1}\{F(s+a)\}, \\
\mathcal{L}\left\{\frac{dy}{dt}\right\} &= s\,\mathcal{L}\{y(t)\} - y(0).
\end{aligned}
$$

(AB 1) An object weighing 5 lbs stretches a spring 4 inches. It is attached to a viscous damper with damping constant 16 lb $\cdot$ sec/ft. The object is pulled down an additional 2 inches and is released with initial velocity 3 ft/sec upwards.

Write the differential equation and initial conditions that describe the position of the object.

(AB 2) A 2-kg object is attached to a spring with constant 80 N/m and to a viscous damper with damping constant β. The object is pulled down to 10cm below its equilibrium position and released with no initial velocity.

Write the differential equation and initial conditions that describe the position of the object.

If $\beta = 20$ N $\cdot$ s/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

If $\beta = 30$ N $\cdot$ s/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

(AB 3) A 3-kg object is attached to a spring with constant k and to a viscous damper with damping constant 42 N$\cdot$sec/m. The object is set in motion from its equilibrium position with initial velocity 5 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object.

If $k = 100$ N/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

If $k = 200$ N/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

(AB 4) A 4-kg object is attached to a spring with constant 70 N/m and to a viscous damper with damping constant β. The object is pushed up to 5cm above its equilibrium position and released with initial velocity 3 m/s downwards.

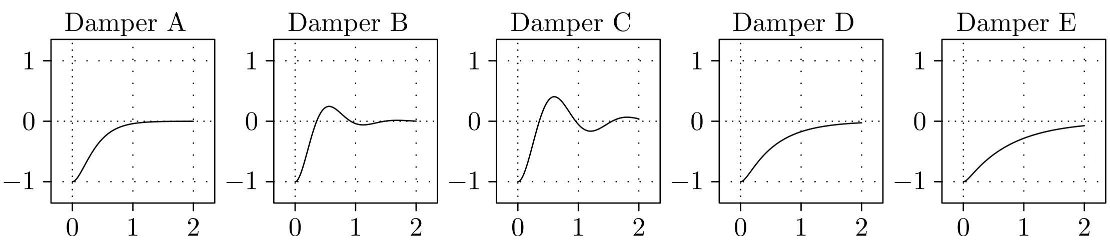
Write the differential equation and initial conditions that describe the position of the object. Then find the value of β for which the system is critically damped. Be sure to include units for β.

(AB 5) An object of mass m is attached to a spring with constant 80 N/m and to a viscous damper with damping constant 20 N·s/m. The object is pulled down to 5cm below its equilibrium position and released with initial velocity 3 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object. Then find the value of m for which the system is critically damped. Be sure to include units for m.

(AB 6) Five objects, each with mass 1 kg, are attached to five springs, each with constant 25 N/m. Five dampers with unknown constants are attached to the objects. In each case, the object is pulled down to a distance 1 cm below the equilibrium position and released from rest. You are given that the system is critically damped in exactly one of the five cases.

Here are the graphs of the objects' positions with respect to time:



(a) For which damper is the system critically damped?
(b) For which dampers is the system overdamped?
(c) For which dampers is the system underdamped?
(d) Which damper has the highest damping constant? Which damper has the lowest damping constant?

(AB 7) Find the general solution to the equation $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t \ln t$ on the interval $0 < t < \infty$.

(AB 8) Find the general solution to the equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-2t} \arctan t$.

(AB 9) Find the general solution to the equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \sin(e^t)$.

(AB 10) Solve the initial-value problem $\frac{d^2y}{dt^2} + 9y = 9\sec^2 3t$, $y(0) = 4$, $y'(0) = 6$, on the interval $-\pi/6 < t < \pi/6$.

(AB 11) The general solution to the differential equation $t^2\frac{d^2y}{dt^2} - 2y = 0$, $t > 0$, is $y(t) = C_1t^2 + C_2t^{-1}$. Solve the initial-value problem $t^2\frac{d^2y}{dt^2} - 2y = 7t^3$, $y(1) = 1$, $y'(1) = 2$ on the interval $0 < t < \infty$.

(AB 12) The general solution to the differential equation $t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} - 3y = 0$, $t > 0$, is $y(t) = C_1t^3 + C_2t^{-1}$. Find the general solution to the differential equation $t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} - 3y = 6t^{-1}$.

(AB 13) Find the general solution to the following differential equations.

(a) $6\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = 3e^{4t}$.
(b) $16\frac{d^2y}{dt^2} - y = e^{t/4}\sin t$.
(c) $\frac{d^2y}{dt^2} + 49y = 3t\sin 7t$.
(d) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 4e^{3t}$.
(e) $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 85y = t\sin(3t)$.
(f) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y = 7e^{-5t}$.
(g) $16\frac{d^2y}{dt^2} - 24\frac{dy}{dt} + 9y = 6t^2 + \cos(2t)$.
(h) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = t^2e^{3t}$.
(i) $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 25y = 3e^{-5t}$.
(j) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 3\cos(2t)$.
(k) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 5\sin(4t)$.
(l) $\frac{d^2y}{dt^2} + 9y = 5\sin(3t)$.
(m) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2t^2$.
(n) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 3t$.
(o) $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 5t + \cos(2t)$.
(p) $\frac{d^2y}{dt^2} - 9y = 2e^t + e^{-t} + 5t + 2$.
(q) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 3e^{2t} + 5\cos t$.

(AB 14) Solve the following initial-value problems. Express your answers in terms of real functions.

(a) $16\frac{d^2y}{dt^2} - y = 3e^t$, $y(0) = 1$, $y'(0) = 0$.
(b) $\frac{d^2y}{dt^2} + 49y = \sin 7t$, $y(\pi) = 3$, $y'(\pi) = 4$.
(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y = 25t^2$, $y(2) = 0$, $y'(2) = 3$.
(d) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 4e^{3t}$, $y(0) = 1$, $y'(0) = 3$.

(AB 15) A 3-kg mass stretches a spring 5 cm. It is attached to a viscous damper with damping constant 27 $\mathrm{N \cdot s/m}$ and is initially at rest at equilibrium. At time $t = 0$, an external force begins to act on the object; at time t seconds, the force is $3\cos(20t)$ N upwards.

Write the differential equation and initial conditions that describe the position of the object.

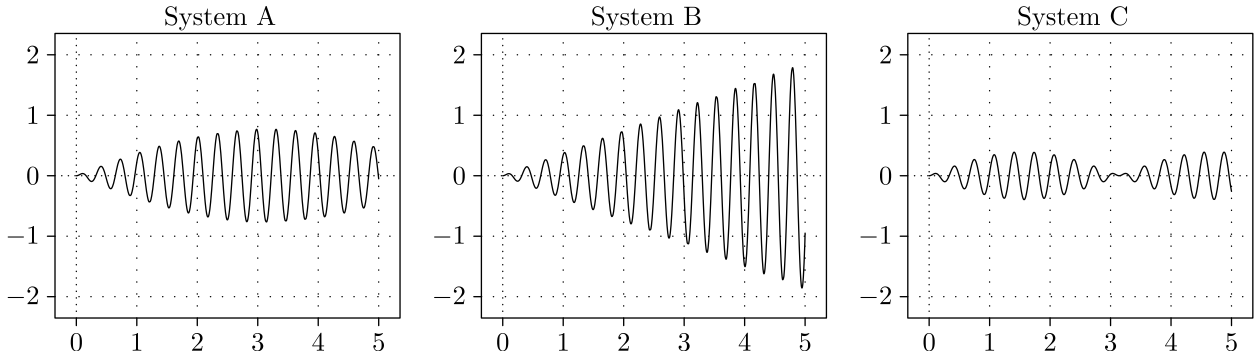
(AB 16) An object weighing 4 lbs stretches a spring 2 inches. It is initially at rest at equilibrium. At time $t = 0$, an external force begins to act on the object; at time t seconds, the force is $3\cos(\omega t)$ pounds, directed upwards. There is no damping.

Write the differential equation and initial conditions that describe the position of the object. Then find the value of ω for which resonance occurs. Be sure to include units for ω.

(AB 17) A 4-kg object is suspended from a spring. There is no damping. It is initially at rest at equilibrium. At time $t = 0$, an external force begins to act on the object; at time t seconds, the force is $7\sin(\omega t)$ newtons, directed upwards.

(a) Write the differential equation and initial conditions that describe the position of the object.
(b) It is observed that resonance occurs when $\omega = 20$ rad/sec. What is the constant of the spring? Be sure to include units.

(AB 18) A 1-kg object is suspended from a spring with constant 400 N/m. There is no damping. It is initially at rest at equilibrium. At time $t = 0$, an external force begins to act on the object; at time t seconds, the force is $15\cos(\omega t)$ pounds, directed upwards. Illustrated are the object's position for three different values of ω. You are given that the three values are $\omega = 18$ radians/second, $\omega = 19$ radians/second, and $\omega = 20$ radians/second. Determine the value of ω that will produce each image.



(AB 19) Using the definition $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$ (**not** the table on the front of the exam), find the Laplace transforms of the following functions.

(a) $f(t) = e^{-11t}$

(b) $f(t) = t$

(c) $f(t) = \begin{cases} 3e^t, & 0 < t < 4, \\ 0, & 4 \leq t. \end{cases}$

(AB 20) Find the Laplace transforms of the following functions. You may use the table on the front of the exam.

(a) $f(t) = t^4 + 5t^2 + 4$

(b) $f(t) = (t+2)^3$

(c) $f(t) = 9e^{4t+7}$

(d) $f(t) = -e^{3(t-2)}$

(e) $f(t) = (e^t + 1)^2$

(f) $f(t) = 8\sin(3t) - 4\cos(3t)$

(g) $f(t) = t^2 e^{5t}$

(h) $f(t) = 7e^{3t}\cos 4t$

(i) $f(t) = 4e^{-t}\sin 5t$

(AB 21) For each of the following problems, find y.

(a) $\mathcal{L}\{y\} = \frac{2s+8}{s^2+2s+5}$

(b) $\mathcal{L}\{y\} = \frac{5s-7}{s^4}$

(c) $\mathcal{L}\{y\} = \frac{s+2}{(s+1)^4}$

(d) $\mathcal{L}\{y\} = \frac{2s-3}{s^2-4}$

(e) $\mathcal{L}\{y\} = \frac{1}{s^2(s-3)^2}$

(f) $\mathcal{L}\{y\} = \frac{s+2}{s(s^2+4)}$

(g) $\mathcal{L}\{y\} = \frac{s}{(s^2+1)(s^2+9)}$

(AB 22) Solve the following initial-value problems using the Laplace transform.

(a) $\frac{dy}{dt} - 9y = \sin 3t,\ y(0) = 1$
(b) $\frac{dy}{dt} - 2y = 3e^{2t},\ y(0) = 2$
(c) $\frac{dy}{dt} + 5y = t^3,\ y(0) = 3$
(d) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 0,\ y(0) = 1,\ y'(0) = 1$
(e) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = e^{-t},\ y(0) = 0,\ y'(0) = 1$
(f) $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = e^{3t},\ y(0) = 2,\ y'(0) = 3$
(g) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = t^2e^{2t},\ y(0) = 3,\ y'(0) = 2$
(h) $\frac{d^2y}{dt^2} - 4y = e^t \sin(3t),\ y(0) = 0,\ y'(0) = 0$
(i) $\frac{d^2y}{dt^2} + 9y = \cos(2t),\ y(0) = 1,\ y'(0) = 5$

Answer key

(Answer 1) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in feet). Then

$$\frac{5}{32}\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 15x = 0, \qquad x(0) = -\frac{1}{6}, \quad x'(0) = 3.$$

(Answer 2) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$2\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + 80x = 0, \qquad x(0) = -0.1, \quad x'(0) = 0.$$

If $\beta = 20$ N$\cdot$s/m, then the system is underdamped, and we do expect to see decaying oscillations.
If $\beta = 30$ N$\cdot$s/m, then the system overdamped, and we do not expect to see decaying oscillations.

(Answer 3) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$3\frac{d^2x}{dt^2} + 42\frac{dx}{dt} + kx = 0, \qquad x(0) = 0, \quad x'(0) = -5.$$

If $k = 100$ N/m, then the system is overdamped, and we do not expect to see decaying oscillations.
If $k = 200$ N/m, then the system underdamped, and we do expect to see decaying oscillations.

(Answer 4) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$4\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + 70x = 0, \qquad x(0) = 0.05, \quad x'(0) = -3.$$

Critical damping occurs when $\beta = 4\sqrt{70}$ N$\cdot$s/m.

(Answer 5) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$m\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 80x = 0, \qquad x(0) = -0.05, \quad x'(0) = -3.$$

Critical damping occurs when $m = \frac{5}{4}$ kg.

(Answer 6)
(a) Damper A.
(b) Dampers D and E.
(c) Dampers B and C.
(d) Damper E has the highest damping constant. Damper C has the lowest damping constant.

(Answer 7) If $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t \ln t$, then $y(t) = c_1 e^t + c_2\, t\, e^t + \frac{1}{2}t^2 e^t \ln t - \frac{3}{4}t^2 e^t$ for all $t > 0$.

(Answer 8) If $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-2t}\arctan t$, then $y(t) = c_1 e^{-2t} + c_2\, t\, e^{-2t} + \frac{1}{2}\big(t^2\arctan t - t\ln(1+t^2) - t + \arctan t\big)$.

(Answer 9) If $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \sin(e^t)$, then $y = c_1 e^{-t} + c_2 e^{-2t} - e^{-2t}\sin(e^t)$.

(Answer 10) If $\frac{d^2y}{dt^2} + 9y = 9\sec^2 3t$, $y(0) = 4$, $y'(0) = 6$, then

$$y(t) = 5\cos(3t) + 2\sin(3t) + (\sin(3t))\ln(\tan(3t) + \sec(3t)) - 1$$

for all $-\pi/6 < t < \pi/6$.

(Answer 11) If $t^2\frac{d^2y}{dt^2} - 2y = 7t^3$, $y(1) = 1$, $y'(1) = 2$, then $y(t) = \frac{7}{4}t^3 - \frac{4}{3}t^2 + \frac{7}{12}t^{-1}$ for all $0 < t < \infty$.

(Answer 12) If $t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} - 3y = 6t^{-1}$, then $y(t) = -\frac{3}{2}t^{-1}\ln t + C_1t^3 + C_2t^{-1}$ for all $t > 0$.

(Answer 13)

(*a*) The general solution to $6\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = 0$ is $y_g = C_1e^{-t/2} + C_2e^{-t/3}$. To solve $6\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = 2e^{4t}$ we make the guess $y_p = Ae^{4t}$. The solution is $y = C_1e^{-t/2} + C_2e^{-t/3} + \frac{1}{39}e^{4t}$.

(*b*) The general solution to $16\frac{d^2y}{dt^2} - y = 0$ is $y_g = C_1e^{t/4} + C_2e^{-t/4}$. To solve $16\frac{d^2y}{dt^2} - y = e^{t/4}\sin t$ we make the guess $y_p = Ae^{t/4}\sin t + Be^{t/4}\cos t$. The solution is $y = C_1e^{t/4} + C_2e^{-t/4} - (1/20)e^{t/4}\sin t - (1/40)e^{t/4}\cos t$.

(*c*) The general solution to $\frac{d^2y}{dt^2} + 49y = 0$ is $y_g = C_1\sin 7t + C_2\cos 7t$. To solve $\frac{d^2y}{dt^2} + 49y = 3t\sin 7t$ we make the guess $y_p = At^2\sin 7t + Bt\sin 7t + Ct^2\cos 7t + Dt\cos 7t$. The solution is $y = C_1\sin 7t + C_2\cos 7t - (3/28)t^2\cos 7t + (3/4)t\sin 7t$.

(*d*) The general solution to $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 0$ is $y_g = C_1 + C_2e^{4t}$. To solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 4e^{3t}$ we make the guess $y_p = Ae^{3t}$. The solution is $y = C_1 + C_2e^{4t} - (4/3)e^{3t}$.

(*e*) The general solution to $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 85y = 0$ is $y_g = C_1e^{-6t}\cos(7t) + C_2e^{-6t}\sin(7t)$. To solve $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 85y = t\sin(3t)$ we make the guess $y_p = At\sin(3t) + Bt\cos(3t) + C\sin(3t) + D\cos(3t)$. The solution is

$$y = C_1e^{-6t}\cos(7t) + C_2e^{-6t}\sin(7t) + \frac{19}{1768}t\sin(3t) - \frac{9}{1768}t\cos(3t) - \frac{1353}{781456}\sin(3t) + \frac{606}{781456}\cos(3t).$$

(*f*) The general solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y = 0$ is $y_g = C_1e^{2t} + C_2e^{-5t}$. To solve $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y = 7e^{-5t}$ we make the guess $y_p = Ate^{-5t}$. The solution is $y = C_1e^{2t} + C_2e^{-5t} - (1/7)te^{-5t}$.

(*g*) The general solution to $16\frac{d^2y}{dt^2} - 24\frac{dy}{dt} + 9y = 0$ is $y_g = C_1e^{(3/4)t} + C_2te^{(3/4)t}$. To solve $16\frac{d^2y}{dt^2} - 24\frac{dy}{dt} + 9y = 6t^2 + \cos(2t)$ we make the guess $y_p = At^2 + Bt + C + D\cos(2t) + E\sin(2t)$. The solution is $y = C_1e^{(3/4)t} + C_2te^{(3/4)t} + (2/3)t^2 + (32/9)t + 64/9 - (48/5329)\cos(2t) - (55/5329)\sin(2t)$.

(*h*) The general solution to $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = 0$ is $y_g = C_1e^{3t}\cos(4t) + C_2e^{3t}\sin(4t)$. To solve $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = t^2e^{3t}$ we make the guess $y_p = At^2e^{3t} + Bte^{3t} + Ce^{3t}$. The solution is $y = C_1e^{3t}\cos(4t) + C_2e^{3t}\sin(4t) + (1/16)t^2e^{3t} - (1/128)e^{3t}$.

(*i*) The general solution to $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 25y = 0$ is $y_g = C_1e^{-5t} + C_2te^{-5t}$. To solve $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 25y = 3e^{-5t}$ we make the guess $y_p = At^2e^{-5t}$. The solution is $y = C_1e^{-5t} + C_2te^{-5t} + (3/2)t^2e^{-5t}$.

(*j*) The general solution to $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$ is $y_g = C_1e^{-2t} + C_2e^{-3t}$. To solve $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 3\cos(2t)$, we make the guess $y_p = A\cos(2t) + B\sin(2t)$. The solution is $y = C_1e^{-2t} + C_2e^{-3t} + \frac{15}{52}\sin(2t) + \frac{3}{52}\cos(2t)$.

(*k*) The general solution to $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0$ is $y_g = C_1e^{-3t} + C_2te^{-3t}$. To solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 5\sin(4t)$, we make the guess $y_p = A\cos(4t) + B\sin(4t)$.

(*l*) The general solution to $\frac{d^2y}{dt^2} + 9y = 0$ is $y_g = C_1\cos(3t) + C_2\sin(3t)$. To solve $\frac{d^2y}{dt^2} + 9y = 5\sin(3t)$, we make the guess $y_p = C_1t\cos(3t) + C_2t\sin(3t)$. The solution is $y = C_1\cos(3t) + C_2\sin(3t) - \frac{5}{6}t\cos(3t)$.

(*m*) The general solution to $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$ is $y_g = C_1e^{-t} + C_2te^{-t}$. To solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2t^2$, we make the guess $y_p = At^2 + Bt + C$. The solution is $y = C_1e^{-t} + C_2te^{-t} + 2t^2 - 8t + 12$.

(*n*) The general solution to $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 0$ is $y_g = C_1 + C_2e^{-2t}$. To solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 3t$, we make the guess $y_p = At^2 + Bt$. The solution is $y = C_1 + C_2e^{-2t} + \frac{3}{4}t^2 - \frac{3}{4}t$.

(*o*) The general solution to $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 0$ is $y_g = C_1e^{3t} + C_2e^{4t}$. To solve $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 5t + \cos(2t)$, we make the guess $y_p = At + B + C\cos(2t) + D\sin(2t)$.

(*p*) The general solution to $\frac{d^2y}{dt^2} - 9y = 0$ is $y_g = C_1e^{3t} + C_2e^{-3t}$. To solve $\frac{d^2y}{dt^2} - 9y = 2e^t + e^{-t} + 5t + 2$, we make the guess $y_p = Ae^t + Be^{-t} + Ct + D$.

(*q*) The general solution to $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 0$ is $y_g = C_1e^{2t} + C_2te^{2t}$. To solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 3e^{2t} + 5\cos t$, we make the guess $y_p = At^2e^{2t} + B\cos t + C\sin t$.

(Answer 14)

(a) If $16\frac{d^2y}{dt^2} - y = 3e^t$, $y(0) = 1$, $y'(0) = 0$, then $y(t) = \frac{1}{5}e^t + \frac{4}{5}e^{-t/4}$.

(b) If $\frac{d^2y}{dt^2} + 49y = \sin 7t$, $y(\pi) = 3$, $y'(\pi) = 4$, then $y(t) = -\frac{1}{14}t\cos(7t) + \frac{\pi-42}{14}\cos(7t) - \frac{55}{98}\sin(7t)$.

(c) If $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y = 25t^2$, $y(2) = 0$, $y'(2) = 0$, then $y(t) = -\frac{5}{2}t^2 - \frac{3}{2}t + \frac{19}{20} + \frac{11}{60}e^{2t-4} - \frac{356}{30}e^{-5t+10}$.

(d) If $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = 4e^{3t}$, $y(0) = 1$, $y'(0) = 3$, then $y(t) = -\frac{4}{3}e^{3t} + \frac{7}{4}e^{4t} + \frac{7}{12}$.

(Answer 15) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters).

Then

$$3\frac{d^2x}{dt^2} + 27\frac{dx}{dt} + 588x = 3\cos(20t), \qquad u(0) = 0, \quad u'(0) = 0.$$

(Answer 16) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in feet). Then

$$\frac{4}{32}\frac{d^2x}{dt^2} + 24x = 3\cos(\omega t), \qquad x(0) = 0, \quad x'(0) = 0.$$

Resonance occurs when $\omega = 8\sqrt{3}\ \mathrm{s}^{-1}$.

(Answer 17)

(a) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in feet). Let k denote the constant of the spring (in N·s/m). Then

$$4\frac{d^2x}{dt^2} + kx = 7\sin(\omega t), \qquad x(0) = 0, \quad x'(0) = 0.$$

(b) The spring constant is $k = 1600$ N·s/m.

(Answer 18) $\omega = 18$ radians/second in Picture C. $\omega = 19$ radians/second in Picture A. $\omega = 20$ radians/second in Picture B.

(Answer 19)

(a) $\mathcal{L}\{t\} = \frac{1}{s^2}$.

(b) $\mathcal{L}\{e^{-11t}\} = \frac{1}{s+11}$.

(c) $\mathcal{L}\{f(t)\} = \frac{3-3e^{4-4s}}{s-1}$.

(Answer 20)

(a) $\mathcal{L}\{t^4 + 5t^2 + 4\} = \frac{96}{s^5} + \frac{10}{s^3} + \frac{4}{s}$.

(b) $\mathcal{L}\{(t+2)^3\} = \mathcal{L}\{t^3 + 6t^2 + 12t + 8\} = \frac{6}{s^4} + \frac{12}{s^3} + \frac{12}{s^2} + \frac{8}{s}$.

(c) $\mathcal{L}\{9e^{4t+7}\} = \mathcal{L}\{9e^7 e^{4t}\} = \frac{9e^7}{s-4}$.

(d) $\mathcal{L}\{-e^{3(t-2)}\} = \mathcal{L}\{-e^{-6}e^{3t}\} = -\frac{e^{-6}}{s-3}$.

(e) $\mathcal{L}\{(e^t+1)^2\} = \mathcal{L}\{e^{2t} + 2e^t + 1\} = \frac{1}{s-2} + \frac{2}{s-1} + \frac{1}{s}$.

(f) $\mathcal{L}\{8\sin(3t) - 4\cos(3t)\} = \frac{24}{s^2+9} - \frac{4s}{s^2+9}$.

(g) $\mathcal{L}\{t^2 e^{5t}\} = \frac{2}{(s-5)^3}$.

(h) $\mathcal{L}\{7e^{3t}\cos 4t\} = \frac{7s-21}{(s-3)^2+16}$.

(i) $\mathcal{L}\{4e^{-t}\sin 5t\} = \frac{20}{(s+1)^2+25}$.

(Answer 21)

(a) If $\mathcal{L}\{y\} = \frac{2s+8}{s^2+2s+5}$, then $y = 2e^{-t}\cos(2t) + 3e^{-t}\sin(2t)$.

(b) If $\mathcal{L}\{y\} = \frac{5s-7}{s^4}$, then $y = \frac{5}{2}t^2 - \frac{7}{6}t^3$.

(c) If $\mathcal{L}\{y\} = \frac{s+2}{(s+1)^4}$, then $y = \frac{1}{2}t^2 e^{-t} + \frac{1}{6}t^3 e^{-t}$.

(d) If $\mathcal{L}\{y\} = \frac{2s-3}{s^2-4}$, then $y = (1/4)e^{2t} + (7/4)e^{-2t}$.

(e) If $\mathcal{L}\{y\} = \frac{1}{s^2(s-3)^2}$, then $y = \frac{2}{27} + \frac{1}{9}t - \frac{2}{27}e^{3t} + \frac{1}{9}t\,e^{3t}$.

(f) If $\mathcal{L}\{y\} = \frac{s+2}{s(s^2+4)}$, then $y = \frac{1}{2} - \frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t)$.

(g) If $\mathcal{L}\{y\} = \frac{s}{(s^2+1)(s^2+9)}$, then $y = \frac{1}{8}\cos t - \frac{1}{8}\cos 3t$.

(Answer 22)

(a) If $\frac{dy}{dt} - 9y = \sin 3t$, $y(0) = 1$, then $y(t) = -\frac{1}{30}\sin 3t - \frac{1}{90}\cos 3t + \frac{31}{30}e^{9t}$.

(b) If $\frac{dy}{dt} - 2y = 3e^{2t}$, $y(0) = 2$, then $y = 3te^{2t} + 2e^{2t}$.

(c) If $\frac{dy}{dt} + 5y = t^3$, $y(0) = 3$, then $y = \frac{1}{5}t^3 - \frac{3}{25}t^2 + \frac{6}{125}t - \frac{6}{625} + \frac{1881}{625}e^{-5t}$.

(d) If $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 0$, $y(0) = 1$, $y'(0) = 1$, then $y(t) = e^{2t} - te^{2t}$.

(e) If $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$, then $y(t) = \frac{1}{5}(e^{-t} - e^t\cos t + 7e^t \sin t)$.

(f) If $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = e^{3t}$, $y(0) = 2$, $y'(0) = 3$, then $y(t) = 4e^{3t} - 2e^{4t} - te^{3t}$.

(g) If $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = t^2e^{2t}$, $y(0) = 3$, $y'(0) = 2$, then $y(t) = -\frac{15}{8}e^{4t} + \frac{39}{8}e^{2t} - \frac{1}{4}te^{2t} - \frac{1}{2}t^2e^{2t} - t^3e^{2t}$.

(h) If $\frac{d^2y}{dt^2} - 4y = e^t\sin(3t)$, $y(0) = 0$, $y'(0) = 0$, then $y(t) = \frac{1}{40}e^{2t} - \frac{1}{72}e^{-2t} - \frac{1}{90}e^t\cos(3t) - \frac{1}{45}e^t\sin(3t)$.

(i) If $\frac{d^2y}{dt^2} + 9y = \cos(2t)$, $y(0) = 1$, $y'(0) = 5$, then $y(t) = \frac{1}{5}\cos 2t + \cos 3t + \frac{5}{3}\sin 3t$.