(Problem 1) In this problem we will find the Fourier transform of $e^{-t|x|^2}$ without using complex analysis. Let $f : \mathbb{R} \mapsto \mathbb{R}$ be given by $f(x) = e^{-tx^2}$.

(a) Find $\widehat{f}(0)$.
(b) Find $f'(x)$.
(c) Find $\widehat{(f')}(y)$ in terms of $\widehat{f}(y)$ in two different ways.
(d) Find $\widehat{f}(y)$.
(e) Let $g : \mathbb{R}^n \mapsto \mathbb{R}$ be given by $g(x) = e^{-t|x|^2}$. What is $\widehat{g}(y)$?

(Problem 2) Let $\operatorname{Re} f$ and $\operatorname{Im} f$ denote the real and imaginary parts of a function.

(a) Show that if $f \in L^1(\mathbb{R}^n)$ is real-valued, then $\operatorname{Re} \widehat{f}$ and $\operatorname{Re} \check{f}$ are even and $\operatorname{Im} \widehat{f}$ and $\operatorname{Im} \check{f}$ are odd.
(b) Show that if $f \in L^1(\mathbb{R}^n)$, if $\operatorname{Re} f$ is even, and if $\operatorname{Im} f$ is odd, then $\widehat{f}$ and $\check{f}$ are both real-valued.

(Problem 3) Let $f \in L^2(\mathbb{R}^n)$.

(a) Find $\widehat{u}$ in terms of $\widehat{f}$ if $-\Delta u + u = f$ in $\mathbb{R}^n$.
(b) Show that $\widehat{u} \in L^2(\mathbb{R}^n)$.
(c) Recall that $u = \widecheck{(\widehat{u})}$. Show that if f is real-valued then u is real-valued.
(d) Is $Du \in L^2(\mathbb{R}^n)$? Is $D^2u \in L^2(\mathbb{R}^n)$? Is $D^3u \in L^2(\mathbb{R}^n)$?