Math 2554H, Fall 2018

Throughout, if you are asked to find a limit, you are to either find a real number equal to the limit, state that the limit is ∞, state that the limit is $-\infty$, or state that the limit does not exist.

If you need it, the following information will be printed on the cover of the exam:

If f is a function, then $f'(x) = \dfrac{d}{dx}(f(x)) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

$\lim\limits_{h\to 0} \dfrac{\sin h}{h} = 1$, $\lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} = 0$, $\sin(x+h) = \sin x \cos h + \sin h \cos x$, $\cos(x+h) = \cos x \cos h - \sin x \sin h$.

The volume of a cylinder of radius r and height h is $\pi r^2 h$.

The volume of a cone of radius r and height h is $\dfrac{1}{3}\pi r^2 h$.

The volume of a sphere of radius r is $\dfrac{4}{3}\pi r^3$.

The surface area of a sphere of radius r is $4\pi r^2$.

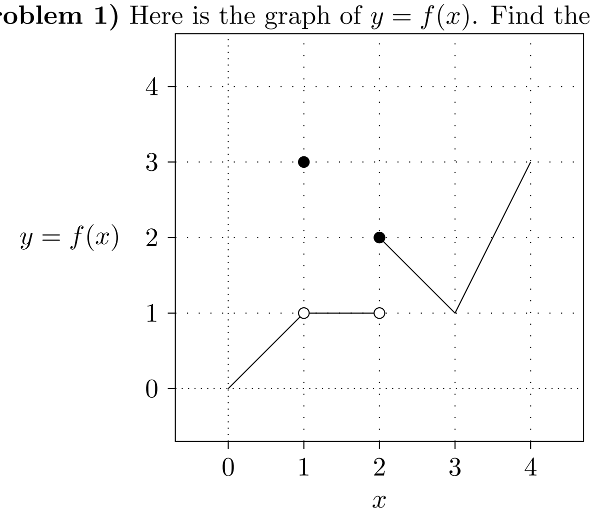
$$\sum_{k=0}^{n-1} r^k = \frac{r^n - 1}{r-1}$$

$$\sum_{k=0}^{n-1} kr^k = \frac{r^n(nr - r - n) + r}{(r-1)^2}.$$

$$\sum_{k=1}^{n} k = \frac{n^2+n}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

(Problem 1) Here is the graph of $y = f(x)$. Find the following values and limits.



(a) $f(1)$

(b) $\lim\limits_{x\to 1} f(x)$

(c) $f(2)$

(d) $\lim\limits_{x\to 2} f(x)$

(e) $\lim\limits_{x\to 2^+} f(x)$

(f) $\lim\limits_{x\to 2^-} f(x)$

(g) $f(3)$

(h) $\lim\limits_{x\to 3} f(x)$

(Problem 2) Here is a table of values of x and of $f(x)$. Approximately how much is $\lim_{x\to 0} f(x)$?

x	$f(x)$
1	1
0.1	0.7177346254
0.01	0.6955550057
0.001	0.6933874626
0.0001	0.6931712038
0.00001	0.6931495828
-0.00001	0.6931447783
-0.0001	0.6931231585
-0.001	0.6929070095
-0.01	0.6907504563
-0.1	0.6696700846
-1	0.5

(Problem 3) Let $f(x) = \dfrac{|x|}{x}$. Complete the following table. What is $\lim_{x\to 0^+} f(x)$? What is $\lim_{x\to 0^-} f(x)$?

x	$f(x)$
1	
0.1	
0.01	
0.001	
-0.001	
-0.01	
-0.1	
-1	

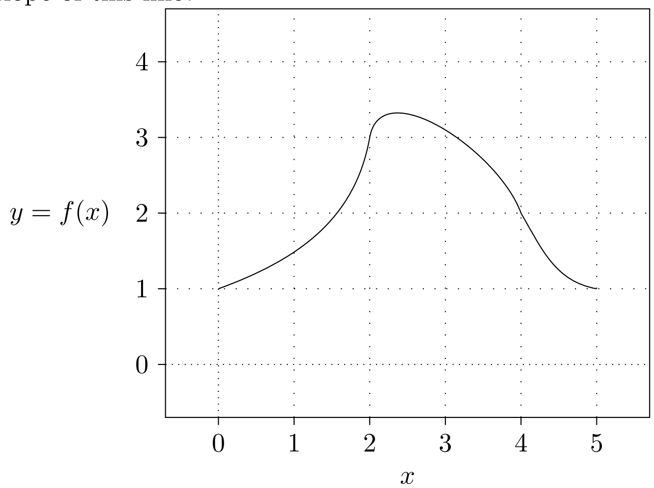
(Problem 4) What is $\lim_{x\to 3} \dfrac{x^3 - 9x^2 + 9x - 27}{x-3}$?

(Problem 5) Suppose that my position at time t is $x(t) = \sqrt{3t+10}$ meters from some fixed base point. Find my average velocity over the interval $[2, 5]$.

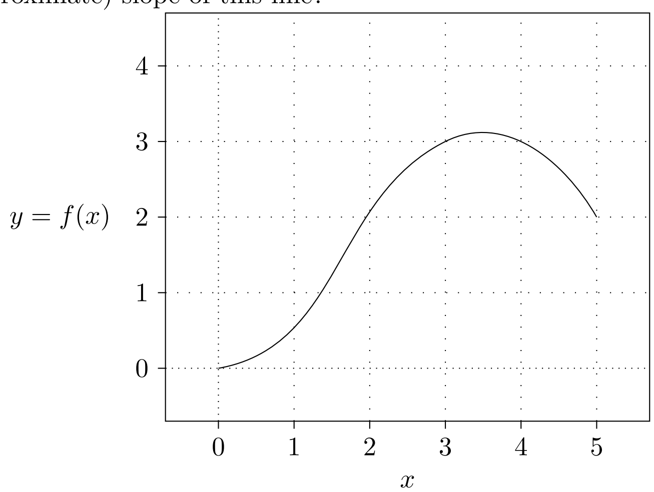
(Problem 6) Suppose that my position at time t is $x(t) = \sqrt{3t+10}$ meters from some fixed base point. Find my instantaneous velocity at time $t = 2$.

(Problem 7) Suppose that my position at time t is $x(t) = \sqrt[3]{t}$ meters from some fixed base point. Find my instantaneous velocity at time $t = 8$.

(Problem 8) Here is the graph of $y = f(x)$. Draw the secant line through $(2, f(2))$ and $(4, f(4))$. What is the slope of this line?



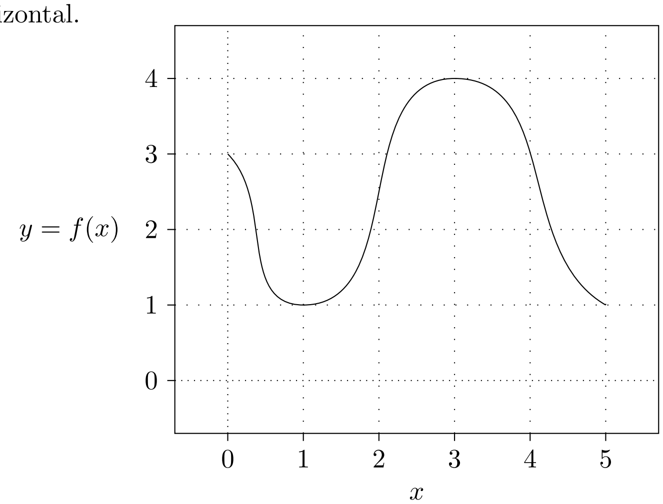
(Problem 9) Here is the graph of $y = f(x)$. Draw the tangent line at the point $(3, f(3))$. What is the (approximate) slope of this line?



(Problem 10) Find the equation for the tangent line to the graph of $y = x^2 - 3x$ at the point $(2, -2)$.

(Problem 11) Find the equation for the tangent line to the graph of $y = \dfrac{18}{x^2}$ at the point $(3, 2)$.

(Problem 12) Here is the graph of $y = f(x)$. Find all the points on this graph at which the tangent line is horizontal.



(Problem 13) Use the squeeze theorem to find $\lim_{x\to 3} x^2 + (x-3)^2 \sin(1/(x-3))$.

(Problem 20) Let $f(x) = \dfrac{2}{(x-2)(x-3)^2(x-4)}$.

(a) Find all numbers a such that $\lim_{x\to a^+} f(x) = \infty$.
(b) Find all numbers a such that $\lim_{x\to a^+} f(x) = -\infty$.
(c) Find all numbers a such that $\lim_{x\to a^-} f(x) = \infty$.
(d) Find all numbers a such that $\lim_{x\to a^-} f(x) = -\infty$.
(e) Find all numbers a such that $\lim_{x\to a} f(x) = \infty$.
(f) Find all numbers a such that $\lim_{x\to a} f(x) = -\infty$.

(Problem 21) Find $\lim_{x\to \pi^+} \dfrac{x^2}{\cos x + 1}$.

(Problem 22) Find $\lim_{x\to 4^+} 3\log_{10}(x-4)$.

(Problem 23) Find $\lim_{x\to\infty} \dfrac{5x^2+4x+2}{3+2x+6x^2}$.

(Problem 24) Find $\lim_{x\to\infty} \dfrac{3+2x+x^2}{2x^3+6x+5}$.

(Problem 25) Find $\lim_{x\to\infty} \sqrt{3 + \dfrac{2}{x^2}}$.

(Problem 26) Find $\lim_{x\to\infty} \arctan x$.

(Problem 27) Find $\lim_{x\to\infty} \operatorname{arccot} x$.

(Problem 28) Find $\lim_{x\to\infty} \operatorname{arcsec} x$.

(Problem 29) Find $\lim_{x\to-\infty} \arctan x$.

(Problem 30) Find $\lim_{x\to-\infty} \operatorname{arccot} x$.

(Problem 31) Find $\lim_{x\to-\infty} \operatorname{arcsec} x$.

(Problem 32) Find all the vertical asymptotes of $f(x) = \dfrac{x+3}{(x+3)(x-3)(x+4)^2}$.

(Problem 33) Find all the horizontal asymptotes of $f(x) = \dfrac{x^3+3x^2+5}{2-x+4x^3}$.

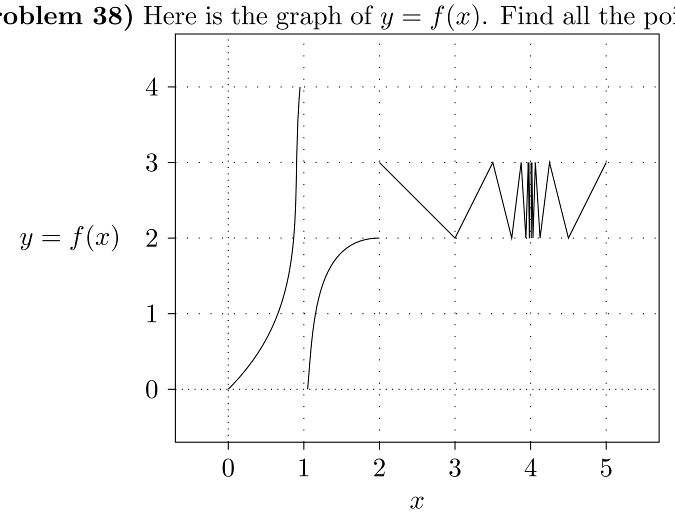
(Problem 34) Find all the horizontal asymptotes of $f(x) = 3 \operatorname{arcsec} x$.

(Problem 35) Find all the slant asymptotes of $f(x) = \dfrac{x^3+3x^2+2x+1}{2x^2+3x+5}$.

(Problem 36) Is $f(x) = x^2 \sin(1/x)$ continuous at $x = 0$? Why or why not?

(Problem 37) Is $f(x) = \arcsin x$ continuous at $x = 1$? Why or why not?

(Problem 38) Here is the graph of $y = f(x)$. Find all the points of discontinuity between $x = 0$ and $x = 5$.



(Problem 39) Sketch a function f that is left continuous at 1 but not continuous at 1.

(Problem 40) Give the intervals of continuity of $y = \arcsin x$, $y = \sqrt{5x-x^2-6}$, $y = \sqrt{x^2-5x+4}$, $y = \dfrac{1}{3x^2+7x+2}$, $y = \lfloor x \rfloor$.

(Problem 41) Let $f(x) = \dfrac{x^2-2x+1}{x-1}$. Let $g(x) = \begin{cases} 1 + x\sin(1/x), & x \neq 0, \\ 1, & x = 0. \end{cases}$

(a) Find $\lim_{x\to 0} g(x)$.

(b) Find $\lim_{x\to 1} f(x)$.

(c) Find $\lim_{x\to 0} f(g(x))$ or state that it does not exist.

(d) Find $\lim_{x\to 0} g(f(x))$ or state that it does not exist.

(Problem 42) Use the intermediate value theorem to show that the equation $x^3 + x = 3$ has a solution. Find an interval of length at most 0.125 containing a solution.

(Problem 43) Use the intermediate value theorem to show that the equation $\cos x - x = 0$ has a solution. Find an interval of length at most $\pi/6$ containing a solution.

(Problem 44) Using εs and δs, write the precise definition of $\lim_{x \to a^+} f(x) = L$.

(Problem 45) Using εs and δs, write the precise definition of $\lim_{x \to a^-} f(x) = L$.

(Problem 46) Using Ms and δs, write the precise definition of $\lim_{x \to a} f(x) = \infty$.

(Problem 47) Using Ms and δs, write the precise definition of $\lim_{x \to a^+} f(x) = \infty$.

(Problem 48) Using Ms and δs, write the precise definition of $\lim_{x \to a^-} f(x) = \infty$.

(Problem 49) Using Ms and δs, write the precise definition of $\lim_{x \to a} f(x) = -\infty$.

(Problem 50) Using Ms and δs, write the precise definition of $\lim_{x \to a^+} f(x) = -\infty$.

(Problem 51) Using Ms and δs, write the precise definition of $\lim_{x \to a^-} f(x) = -\infty$.

(Problem 52) Using εs and Ns, write the precise definition of $\lim_{x \to \infty} f(x) = L$.

(Problem 53) Using εs and Ns, write the precise definition of $\lim_{x \to -\infty} f(x) = L$.

(Problem 54) Using Ms and Ns, write the precise definition of $\lim_{x \to \infty} f(x) = \infty$.

(Problem 55) Using Ms and Ns, write the precise definition of $\lim_{x \to \infty} f(x) = -\infty$.

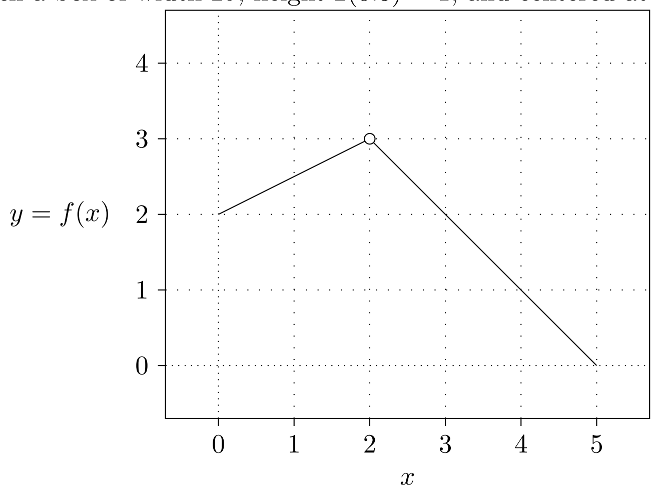
(Problem 56) Using Ms and Ns, write the precise definition of $\lim_{x \to -\infty} f(x) = \infty$.

(Problem 57) Using Ms and Ns, write the precise definition of $\lim_{x \to -\infty} f(x) = -\infty$.

(Problem 58) Find a δ such that, if $|x - 3| < \delta$, then $|x^2 - 9| < 1$.

(Problem 59) Find a δ such that, if $|x - 2| < \delta$, then $|x^2 + 4x - 8| < 5$.

(Problem 60) Here is the graph of $y = f(x)$. Find a δ such that, if $0 < |x - 2| < \delta$, then $|f(x) - 3| < 0.5$. Sketch a box of width 2δ, height $2(0.5) = 1$, and centered at $(2, 3)$.

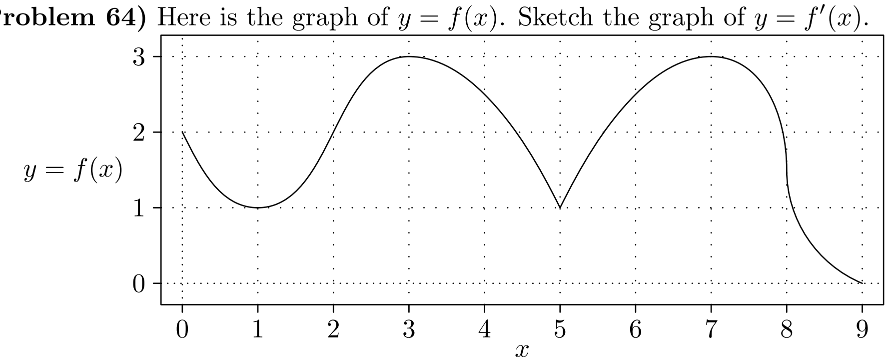


(Problem 61) Let $f(x) = \cos(1/x)$. Show that 0 is *not* $\lim_{x\to 0} f(x)$ by choosing an $\varepsilon > 0$ and showing that, for every $\delta > 0$, there is some x with $0 < |x - 0| < \delta$ and with $|f(x) - 0| \geq \varepsilon$.

(Problem 62) Let $f(x) = \arctan(1/x)$. Show that $\pi/2$ is *not* $\lim_{x\to 0} f(x)$ by choosing an $\varepsilon > 0$ and showing that, for every $\delta > 0$, there is some x with $0 < |x - 0| < \delta$ and with $|f(x) - \pi/2| \geq \varepsilon$.

(Problem 63) Let $f(x) = \dfrac{1}{1 + 2^{1/x}}$. Show that 1 is *not* $\lim_{x\to 0} f(x)$ by choosing an $\varepsilon > 0$ and showing that, for every $\delta > 0$, there is some x with $0 < |x - 0| < \delta$ and with $|f(x) - 1| \geq \varepsilon$.

(Problem 64) Here is the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$.



(Problem 65) Use the limit definition of derivative to find $f'(1)$, where $f(x) = 3x + 2$.

(Problem 66) Use the limit definition of derivative to find $\dfrac{d}{dx}(x^2)\Big|_{x=3}$.

(Problem 67) Find the tangent line to the graph of $y = x^2$ at the point $(3, 9)$.

(Problem 68) Use the limit definition of derivative to find $g'(2)$, where $g(x) = x^3$.

(Problem 69) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt{x})\Big|_{x=25}$.

(Problem 70) Find the tangent line to the graph of $y = \sqrt{x}$ at the point $(25, 5)$.

(Problem 71) Use the limit definition of derivative to find $h'(2)$, where $h(x) = \dfrac{1}{x}$.

(Problem 72) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt[3]{x})\big|_{x=0}$.

(Problem 73) Use the limit definition of derivative to find $f'(x)$, where $f(x) = 3x + 2$.

(Problem 74) Use the limit definition of derivative to find $\dfrac{d}{dx}(x^2)$.

(Problem 75) Use the limit definition of derivative to find $g'(x)$, where $g(x) = x^3$.

(Problem 76) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt{x})$.

(Problem 77) Use the limit definition of derivative to find $h'(x)$, where $h(x) = \dfrac{1}{x^2}$.

(Problem 78) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sin x)$.

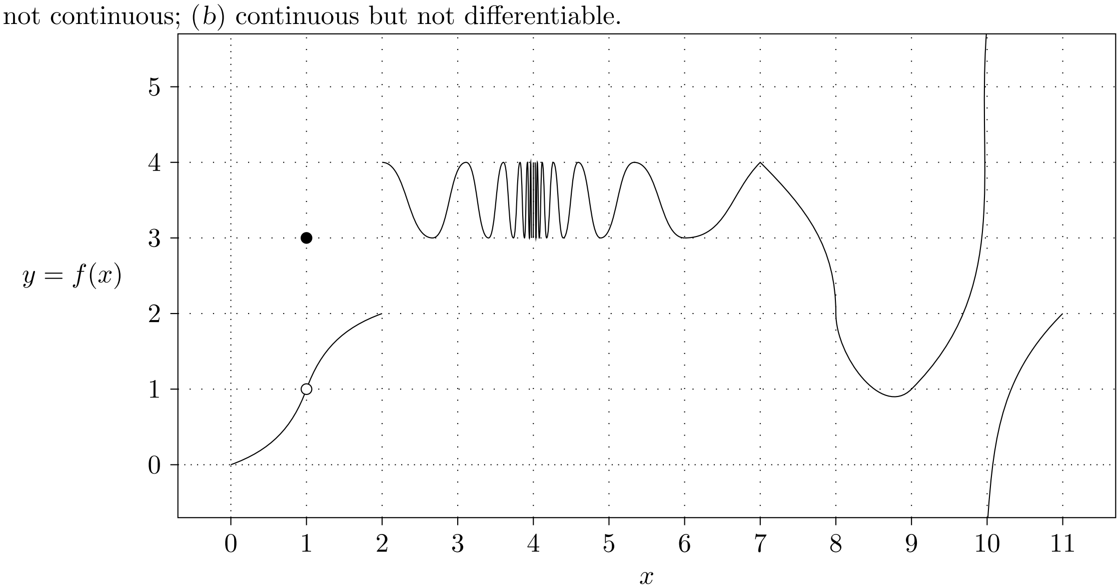
(Problem 79) Use the limit definition of derivative to find $\dfrac{d}{dx}(\cos x)$.

(Problem 80) Use the limit definition of derivative to find $\dfrac{d}{dx}(3^x)$. Your final answer may involve a limit as long as that limit does not depend on x.

(Problem 81) Use the limit definition of derivative to find $\dfrac{d}{dx}(\log_3 x)$. Your final answer may involve a limit as long as that limit does not depend on x.

(Problem 82) Let $f(x) = \dfrac{x^3 - 2x^2}{x - 2}$. What is $f'(2)$?

(Problem 83) Here is the graph of $y = f(x)$. Find all points x with $0 < x < 11$ at which the graph is (a) not continuous; (b) continuous but not differentiable.



(Problem 84) Let $f(x) = x^7 + 9x^3 + e^x$. Find $f'(x)$.

(Problem 85) Find $\dfrac{d}{dx}(3x^3 - 9e^x)$.

(Problem 86) Find $\dfrac{d}{dx}(3\sin x + 4\cos x)$.

(Problem 87) Find $\dfrac{d}{dx}(2\tan x - 3\csc x)$.

(Problem 88) Let $g(x) = \dfrac{3}{x^7} - 2\sqrt[5]{x}$. Find $g'(x)$, $g''(x)$, and $g^{(3)}(x)$.

(Problem 89) Find $\dfrac{d}{dx}(x^4 e^x)$.

(Problem 90) Find $\dfrac{d}{dx}(x^4 \cos x)$.

(Problem 91) Find $\dfrac{d}{dx}(x^5 e^x \cot x)$.

(Problem 92) Find $\dfrac{d}{dx}\left(\dfrac{x^5}{e^x + 1}\right)$.

(Problem 93) Find $\dfrac{d}{dx}\left(\dfrac{e^x}{x^2 + 1}\right)$.

(Problem 94) Find $\dfrac{d}{dx}\left(\dfrac{x^2 \sin x}{2e^x + x}\right)$.

(Problem 95) Find $\dfrac{d^2}{dx^2}(\sec x)$.

(Problem 96) Find $f''(x)$, where $f(x) = \csc x$.

(Problem 97) Use the quotient rule to find $\dfrac{d}{dx}(\tan x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 98) Use the quotient rule to find $\dfrac{d}{dx}(\cot x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 99) Use the quotient rule to find $\dfrac{d}{dx}(\sec x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 100) Use the quotient rule to find $\dfrac{d}{dx}(\csc x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 101) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\cos 3x - 1}{x}$.

(Problem 102) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\sin 5x}{x}$.

(Problem 103) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\tan 5x}{x}$.

(Problem 104) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\sec 2x - 1}{x}$.

(Problem 105) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\cos^2 x - 1}{x}$.

(Problem 106) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{x}$.

(Problem 107) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 2} \dfrac{\sin(x-2)}{4-x^2}$.

(Problem 108) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$. *Hint*: Multiply by $\dfrac{1+\cos x}{1+\cos x}$.

(Problem 109) Using substitutions (not l'Hôpital's rule), find $\lim\limits_{x\to 0} \dfrac{\tan 3x}{\sin 2x}$.

(Problem 110)

(a) Use the squeeze theorem to find $\lim\limits_{x\to \pi/2^-} \dfrac{\cos x}{x-\pi/2}$.

(b) Use the squeeze theorem to find $\lim\limits_{x\to \pi/2^-} \dfrac{1-\sin x}{x-\pi/2}$.

(Problem 112)

(a) Use the squeeze theorem to find $\lim\limits_{x\to \pi^+} \dfrac{\sin x}{x-\pi}$.

(b) Use the squeeze theorem to find $\lim\limits_{x\to \pi^+} \dfrac{\cos x + 1}{x-\pi}$.

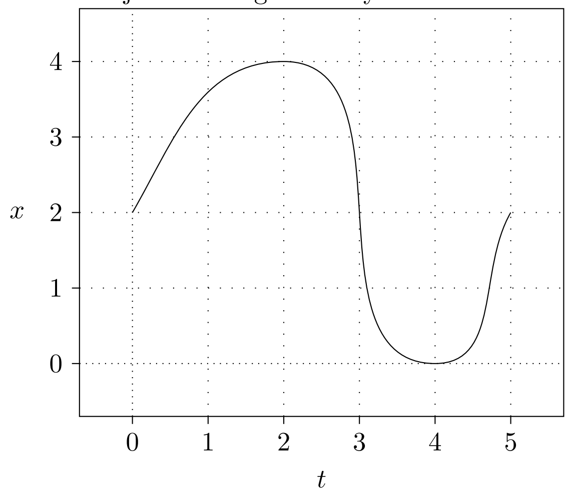
(Problem 114)

(a) Use the squeeze theorem to find $\lim\limits_{x\to 3\pi/2^-} \dfrac{\cos x}{x-3\pi/2}$.

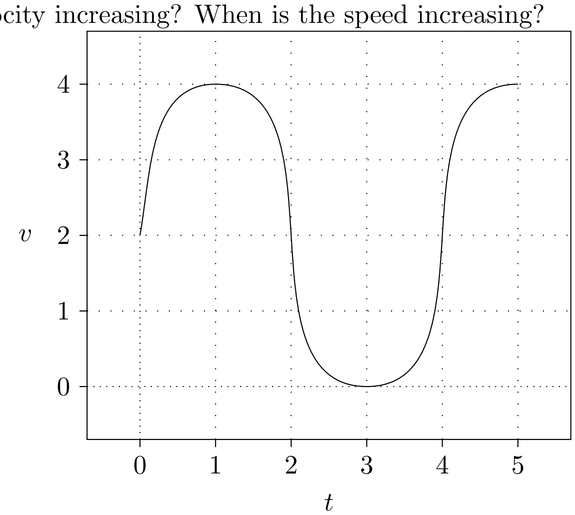
(b) Use the squeeze theorem to find $\lim\limits_{x\to 3\pi/2^-} \dfrac{1+\sin x}{x-3\pi/2}$.

(Problem 116) Suppose that after t seconds, a thrown projectile's height is $h(t) = 4 + 19t - 5t^2$ meters. What is its average velocity over the interval $[1, 3]$? What is the object's maximum height? What is the object's instantaneous velocity at the moment that it hits the ground?

(Problem 117) The following graph shows the displacement of an object to the right of a fixed basepoint as a function of time. When is the object stationary? When is it moving left? When is it moving right? What is the object's average velocity over the interval $[2, 5]$?



(Problem 118) The following graph shows the velocity of an object as a function of time. When is the velocity increasing? When is the speed increasing?



(Problem 119) In 2015, the birth rate in a certain city was 5000 births/year and the death rate was 250 deaths/month. Neglecting immigration and emigration, what was the rate of change of population of the city? Be sure your answer includes units.

(Problem 120) Find $\dfrac{d}{dx}(\csc(7x))$.

(Problem 121) Find $\dfrac{d}{dx}(e^{3x})$.

(Problem 122) Find $\dfrac{d}{dx}(\tan(2x+5))$.

(Problem 123) Find $\dfrac{d}{dx}(e^{4x-3})$.

(Problem 124) Find $\dfrac{d}{dx}(\sin(x^3))$.

(Problem 125) Find $\dfrac{d^2}{dx^2}(\cos(x^4))$.

(Problem 126) Find $\dfrac{d}{dx}((\tan x)^4)$.

(Problem 127) Find $\dfrac{d}{dx}(\sqrt{x^3+5})$.

(Problem 128) Find $\dfrac{d}{dx}(\sqrt[3]{2x-4})$.

(Problem 129) Find $\dfrac{d^2}{dx^2}(\tan x)$.

(Problem 130) Find $f''(x)$, where $f(x) = \cot x$.

(Problem 131) Find $\dfrac{d}{dx}(e^{-x^2})$.

(Problem 132) Find $\dfrac{d}{dx}(\sec(e^x))$.

(Problem 133) Find $\dfrac{d}{dx}(\sin(e^{x^3}))$.

(Problem 134) Use the following table to find $\dfrac{d}{dx}(f(g(x)))\Big|_{x=4}$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	25	6	7
2	3	3	4	2
3	2	5	3	16
4	5	15	1	8
5	1	9	5	4
6	6	27	2	14

(Problem 135) Use the following tables to find $\dfrac{dy}{dx}\Big|_{x=2}$.

x	u	$\dfrac{du}{dx}$
1	2	8
2	3	14
3	1	2
4	4	7
5	6	16
6	5	4

u	y	$\dfrac{dy}{du}$
1	5	15
2	1	3
3	3	5
4	4	9
5	6	27
6	2	25

(Problem 136) Suppose that y satisfies $y^5 + x^4y^3 = x^7$. Find $\dfrac{dy}{dx}$.

(Problem 137) Suppose that y satisfies $\sin(x) + \sin(y) = y$. Find $\dfrac{d^2y}{dx^2}$.

(Problem 138) Find the tangent line to the hyperbola $x^2 - y^2 = 9$ at the point $(5, 4)$.

(Problem 139) Find $\frac{d}{dx}(\ln x)$ using implicit differentiation. Simplify your answer as much as possible.

(Problem 140) Find $\frac{d}{dx}(\log_5 |x|)$.

(Problem 141) Find $\frac{d}{dx}(\ln(x^2 + 1))$.

(Problem 142) Find $\frac{d}{dx}(9^x)$.

(Problem 143) Find $\frac{d}{dx}(x^{\cos x})$.

(Problem 144) Let $y = \dfrac{x^3 e^{3x} \cos^5 x}{(x^2+1)^{18}(e^x+2)^3}$. Find $\frac{dy}{dx}$. If it is notationally simpler to express your answer using y as well as x, do so.

(Problem 145) Find $\frac{d}{dx}(\arcsin x)$ using implicit differentiation. Simplify your answer as much as possible. For what x-values is your formula valid?

(Problem 146) Find $\frac{d}{dx}(\arccos x)$ using implicit differentiation. Simplify your answer as much as possible. For what x-values is your formula valid?

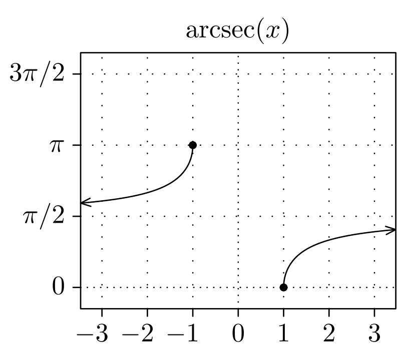
(Problem 147) Find $\frac{d}{dx}(\arctan x)$ using implicit differentiation. Simplify your answer as much as possible.

(Problem 148) Find $\frac{d}{dx}(\operatorname{arccot} x)$ using implicit differentiation. Simplify your answer as much as possible.

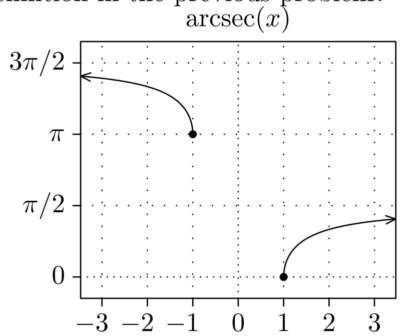
(Problem 149) Find $\frac{d}{dx}(\operatorname{arcsec} x)$, $x \geq 1$, using implicit differentiation. Simplify your answer as much as possible.

(Problem 150) Find $\frac{d}{dx}(\operatorname{arccsc} x)$, $x \geq 1$, using implicit differentiation. Simplify your answer as much as possible.

(Problem 151) I define $\operatorname{arcsec} x$ to be given by the following graph. If $x \leq 1$, is $\frac{d}{dx}(\operatorname{arcsec} x)$ positive or negative?



(Problem 152) I define $\operatorname{arcsec} x$ to be given by the following graph. If $x \leq 1$, is $\dfrac{d}{dx}(\operatorname{arcsec} x)$ positive or negative? Why might I prefer the definition in the previous problem?



(Problem 153) Let $f(x) = x^3 + 3x^5 + 2$. Notice that $f(1) = 6$. Find $\dfrac{d}{dx}(f^{-1}(x))\Big|_{x=6}$.

(Problem 154) Here is a table showing some values of x, $f(x)$, and $f'(x)$. Find $(f^{-1})'(3)$.

x	$f(x)$	$f'(x)$
1	-1	1
2	0	4
3	2	0
4	3	2
5	4	5
6	7	8

(Problem 155) A spherical snowball melts (loses volume) at a rate proportional to its surface area. Show that the rate of change of the radius of the snowball is constant.

(Problem 156) A small planet moves along the ellipse $\left(\dfrac{x-25}{65}\right)^2 + \left(\dfrac{y}{60}\right)^2 = 1$. When the planet's x-coordinate is 77, the x-component of its velocity is 3 units/second. What is the y-component of its velocity?

(Problem 157) A (simplified) hourglass is in the shape of a cone with height 10 cm and radius 6 cm, with the vertex pointed downwards, so that the (remaining) sand is also in the shape of a cone. Sand flows out of the cone at a rate of 3 cm^3/sec. What is the rate of change of the height of the sand left behind at the moment when the height is 4 cm?

(Problem 158) I fly a kite. My kite is ascending at a rate of 20 ft/sec. My kite is also caught in a strong wind that blows it westward at a rate of 30 ft/sec. When my kite is 300 feet above me and 400 feet west of me, how fast is the string running through my fingers? (Assume that the string is always a straight line between me and the kite.)

(Problem 159) A 50-ft ladder rests on the (horizontal) ground and leans on a (vertical) wall. The bottom of the ladder is pulled away from the wall at a rate of 3 ft/sec. How fast is the top of the ladder moving at the moment when the bottom of the ladder is 14 ft from the wall?

(Problem 160) A lighthouse is 300 m from the closest point to a long, straight wall built on shore. The focused beam of its light revolves three times per minute, and when it is aimed at land, it illuminates the wall. How fast is the illuminated point traveling when it passes by a window that is 340 m from the lighthouse?

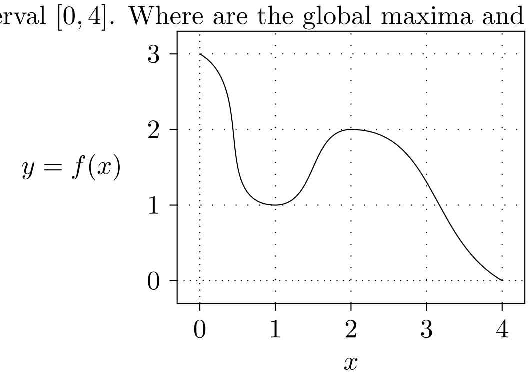
(Problem 161) I watch a spaceship being launched from a distance of 5 km. The ship launches straight up. I measure the line-of-sight angle between the rocket and the ground. At the moment when the angle between the rocket and the ground is $\pi/6$ radians, the angle is changing at a rate of 2 radians/second. How fast is the rocket moving?

(Problem 162) Sketch the graph of a continuous function that has an absolute maximum and a local minimum but no absolute minimum on the interval $(0, 3)$.

(Problem 163) Sketch the graph of a continuous function that has a local maximum value at $x = 2$ where $f'(2) = 0$.

(Problem 164) Sketch the graph of a continuous function that has a local minimum value at $x = 2$ where $f'(2)$ is undefined.

(Problem 165) Here is the graph of $y = f(x)$. Find all the local minima and local maxima of f on the interval $[0, 4]$. Where are the global maxima and minima?



(Problem 166) Find the critical point or points of the function $f(x) = 3x^2 - 6x + 9$.

(Problem 167) Find the critical point or points of the function $f(x) = x^3 - 3x^2 + 9x + 1$.

(Problem 168) Find the critical point or points of the function $f(x) = \sqrt[5]{3x - 2}$.

(Problem 169) Find the absolute maximum and minimum values of the function $f(x) = x^2 - 3x + 5$ on the interval $[1, 4]$ or state that they do not exist.

(Problem 170) Find the absolute maximum and minimum values of the function $f(x) = 32\sqrt{x} - x^2$ on the interval $[0, 9]$ or state that they do not exist.

(Problem 171) Find the absolute maximum and minimum values of the function $f(x) = \dfrac{1}{x^2} + 16x$ on the interval $(0, 3)$ or state that they do not exist.

(Problem 172) Does the Mean Value Theorem apply to the function $f(x) = \sqrt{x}$ on the interval $[1, 9]$? If so, find the point guaranteed to exist by the Mean Value Theorem.

(Problem 173) Does the Mean Value Theorem apply to the function $f(x) = \sqrt{x}$ on the interval $[0, 4]$? If so, find the point guaranteed to exist by the Mean Value Theorem.

(Problem 174) Does the Mean Value Theorem apply to the function $f(x) = \dfrac{1}{x}$ on the interval $[0, 4]$? If so, find the point guaranteed to exist by the Mean Value Theorem.

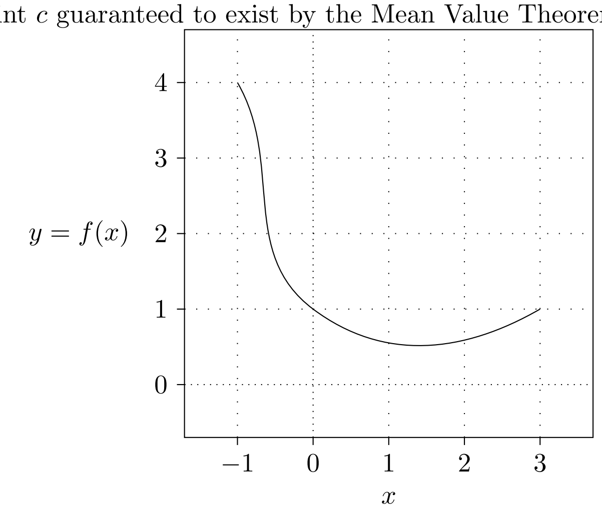
(Problem 175) Does the Mean Value Theorem apply to the function $f(x) = |x|$ on the interval $[-2, 3]$? If so, find the point guaranteed to exist by the Mean Value Theorem.

(Problem 176)
(a) Do the conditions of the Mean Value theorem hold for the function $f(x) = x^3$ on the interval $[-1, 1]$?
(b) Does the conclusion hold?

(Problem 177)
(a) Do the conditions of the Mean Value theorem hold for the function $f(x) = \sqrt[3]{x}$ on the interval $[-1, 1]$?
(b) Does the conclusion hold?

(Problem 178)
(a) Do the conditions of the Mean Value theorem hold for the function $f(x) = x^{2/3}$ on the interval $[-1, 1]$?
(b) Does the conclusion hold?

(Problem 179) Here is the graph of $y = f(x)$ on the interval $[a, b] = [-1, 3]$. By visual inspection, find the point c guaranteed to exist by the Mean Value Theorem. (There may be more than one such point).



(Problem 180) A police department has no speedometers. However, they have a number of traffic cameras and can record the license plate and transit time of every car that passes through a given point. Explain a way that the police department can catch (at least some of the) people who exceed the speed limit over a given stretch of road.

(Problem 181) A rocket ship is stationary just before it is launched. Four minutes after the ship is launched, it is moving at a rate of 12 km/sec. One hour after launch, the ship is in free-fall (and feels no acceleration).
(a) Show that at some point, the crew felt an acceleration of 50 m/sec.
(b) Show that there were at least two points when the crew felt an acceleration of 30 m/sec.

(Problem 182) Sketch a function that is continuous on $[0, 5]$, where $f' > 0$ on $(0, 2)$, $f' < 0$ on $(2, 4)$, and $f' > 0$ on $(4, 5)$.

(Problem 183) Sketch a function that is continuous on $[0, 5]$ and satisfies $f' > 0$ on $(0, 2)$, $f'(2) = 0$, and $f' > 0$ on $(2, 5)$.

(Problem 184) Find the intervals on which $f(x) = 2x^4 - x^2$ is increasing and the intervals on which it is decreasing.

(Problem 185) Find the intervals on which $f(x) = x^4 - 8x^3 + 18x^2 + 5$ is increasing and the intervals on which it is decreasing.

(Problem 186) Find all the critical points of $f(x) = 5x^3 - 3x^5$. Use the First Derivative Test to identify the local maxima and minima.

(Problem 187) Find all the critical points of $f(x) = x^5 - 5x^4 + 5x^3$. Use the First Derivative Test to identify the local maxima and minima.

(Problem 188) Sketch a function that is continuous on $[0, 5]$, where $f' > 0$ on $(0, 1)$, $f' < 0$ on $(1, 3)$, and $f' > 0$ on $(3, 5)$, and where $f'' < 0$ on $(0, 2)$ and $f'' > 0$ on $(2, 5)$.

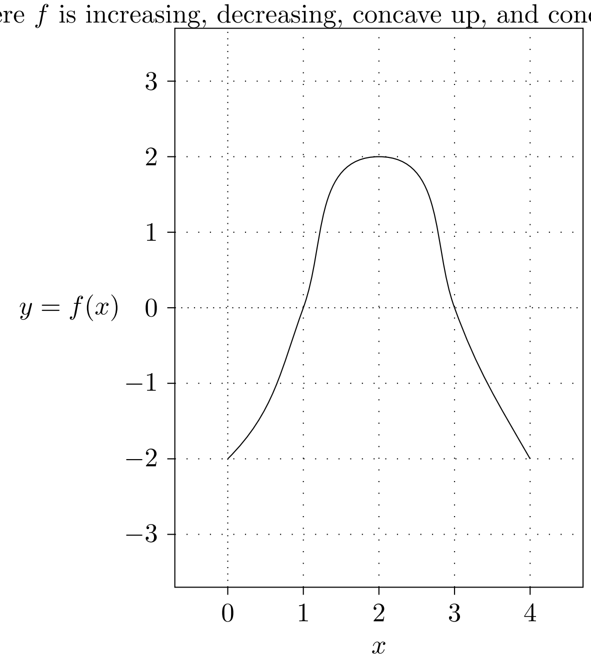
(Problem 189) Sketch a function that is continuous on $[0, 4]$, where $f' > 0$ on $(0, 2)$ and $f' < 0$ on $(2, 4)$, and where $f'' > 0$ on $(0, 2)$ and $f'' > 0$ on $(2, 4)$. Is it possible to sketch such an f that is differentiable everywhere?

(Problem 190) Where is $f(x) = 2x^4 - 2x^3 + 4$ concave up and concave down? What are the inflection points?

(Problem 191) Find the critical points of the function $f(x) = x^3 - 6x^2$. Use the second derivative test to identify which points are local maxima and which are local minima.

(Problem 192) Find the critical points of the function $f(x) = 2x^4 - 3x^6$. What does the second derivative test tell you about these critical points?

(Problem 193) Here is the graph of $y = f'(x)$. (This is **not** the graph of $y = f(x)$!) Find the intervals where f is increasing, decreasing, concave up, and concave down.



(Problem 194) Suppose that $f'(x) = (x - 2)^2(x - 3)^3$. Sketch a possible graph of f.

(Problem 195) Suppose that $f'(x) = \cos(\pi x^2)$. Sketch a possible graph of f on $[-2, 2]$.

(Problem 196) Sketch the graph of $y = \dfrac{x^2 + 12}{3x + 1}$.

(Problem 197) Sketch the graph of $y = x^{2/3} - x$.

(Problem 198) Sketch the graph of $y = \sqrt[3]{x-2} - x$.

(Problem 199) Sketch the graph of $y = \dfrac{1}{\pi} \arccos x$.

(Problem 200) Sketch the graph of $y = \dfrac{1}{\pi} \operatorname{arcsec} x$.

(Problem 201) What two positive real numbers whose product is 70 have the smallest possible sum?

(Problem 202) A rancher plans to make three identical and adjacent rectangular pens against a barn, each with an area of 25 m^2. What are the dimensions of each pen that minimize the amount of fence that must be used?

(Problem 203)

(*a*) A rectangle is constructed with its base on the $x$axis and two of its vertices on the parabola $y = 64 - x^2$. What are the dimensions of the rectangle with the maximum area?

(*b*) Suppose that the rectangle is constrained to lie in the upper half-space. What are the dimensions of the rectangle with the maximum area?

(Problem 204) A cylindrical tank with a circular base and an open top is to be constructed out of 300π square meters of sheet steel. What are the dimensions of the largest possible tank?

(Problem 205) An island is 6 miles from the nearest point on a straight shoreline; that point is 10 miles from a rest area on shore.

(*a*) An athlete can bicycle at a rate of 7.8 miles/hour and swim at a rate of 3 miles/hour. She wants to get from the rest area to the island as quickly as possible. How far should she bicycle before she starts swimming?

(*b*) Another athlete can swim at a rate of 4.5 miles/hour and walk at a rate of 5.1 miles/hour. How far should he walk before he starts to swim to minimize the travel time?

(Problem 206) I want to make a rectangular box by cutting square corners out of an 8.5" by 11" piece of paper and gluing adjacent cut edges. What should the height of the box (that is, the side-length of the removed squares) be to maximize the volume of the box?

(Problem 207) I have \$800 available to fence in a rectangular garden. The fencing for the side of the garden facing the road costs \$15 per foot, and the fencing for the other three sides costs \$5 per foot. Find the dimensions of the garden with the largest possible area.

(Problem 208) A printed page is to contain 18 in^2 of text. The left and right margins of the page are to be 1 inch, and the top and bottom margins of the page are to be 2 inches. Find the dimensions of the smallest page that can contain the given text.

(Problem 209) An 8 foot fence is 3 feet away from a wall that is 20 feet tall. What is the length of the shortest straight ladder that can reach from the ground on the outside of the fence to the wall?

(Problem 210) A 6 foot fence is 5 feet away from a wall that is 10 feet tall. What is the length of the shortest straight ladder that can reach from the ground on the outside of the fence to the wall?

(Problem 211) You are standing on the shore of a circular pond with a radius of 1 km. You want to get to a point directly across the pond. You can walk at a speed of 5 km/hr and swim at a speed of 3 km/hour. What is the fastest way to cross the pond? (Assume that you always swim in straight lines through the water and that you always walk along the edge of the pond.)

(Problem 212) I take three 2 inch wide strips of metal and attach the long edges to form a trapezoidal gutter with an open top and three closed sides each 2 inches wide. What angle should the strips be attached in order to construct a gutter with maximal area?

(Problem 213) Using a linear approximation, estimate $\ln(0.97)$.

(Problem 214) Using a linear approximation, estimate $\arctan(1.01) - \pi/4$.

(Problem 215) Using a linear approximation, estimate $(10.2)^4$.

(Problem 216) Using a linear approximation, estimate $\sqrt[3]{997}$.

(Problem 217) Evaluate $\displaystyle\lim_{x\to\infty} \frac{x + 3\sin x}{2x}$.

(Problem 218) Evaluate $\displaystyle\lim_{x\to\infty} x - \sqrt{x^2 - 3x + 2}$.

(Problem 219) Evaluate $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{\sin x}$.

(Problem 220) Evaluate $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x}$.

(Problem 221) Evaluate $\displaystyle\lim_{x\to 0} \frac{e^x - 1 - x}{\sin x}$.

(Problem 222) Evaluate $\displaystyle\lim_{x\to\infty} x(\pi/2 - \arctan x)$.

(Problem 223) Evaluate $\displaystyle\lim_{x\to\infty} \frac{e^x}{x^2}$.

(Problem 224) Evaluate $\displaystyle\lim_{x\to 0^+} x^{\sin x}$.

(Problem 225) Evaluate $\displaystyle\lim_{x\to 1} \frac{x^2 - 3x + 2}{\ln x}$.

(Problem 226) Evaluate $\displaystyle\lim_{x\to 0^+} (\cos x)^{1/x^2}$.

(Problem 227) Evaluate $\displaystyle\lim_{x\to 1^-} \frac{\sqrt{1-x}}{\arccos x}$.

(Problem 228) Find $\displaystyle\int 9x^7 - 2x^3 + 3\,dx$.

(Problem 229) Find $\displaystyle\int \frac{2}{x} - \frac{3}{x^2} + \sqrt{x} - \frac{4}{\sqrt[3]{x}}\,dx$.

(Problem 230) Find $\displaystyle\int 3\cos 2x - 5\sec^2 2x\,dx$.

(Problem 231) Find $\displaystyle\int 3e^{2x}\,dx$.

(Problem 232) Find $\displaystyle\int \frac{1}{\sqrt{1-9x^2}}\,dx$.

(Problem 233) Find $\displaystyle\int \frac{x^2+3}{x}\,dx$.

(Problem 234) Find $\displaystyle\int \frac{1}{x\sqrt{x^2-9}}\,dx$.

(Problem 235) Find $\displaystyle\int \frac{1}{25+16x^2}\,dx$.

(Problem 236) Solve the initial value problem $F'(x) = e^{3x}$, $F(0) = 5$.

(Problem 237) Solve the initial value problem $F''(x) = 3\sin(2x)$, $F(0) = 1$, $F'(0) = 3$.

(Problem 238) An object's acceleration after t seconds is $-10+10e^{-t/5}$ meters/second^2. Its initial position is $s(0) = 70$ meters and its initial velocity is $v(0) = 0$ meters/second. Find its position function.

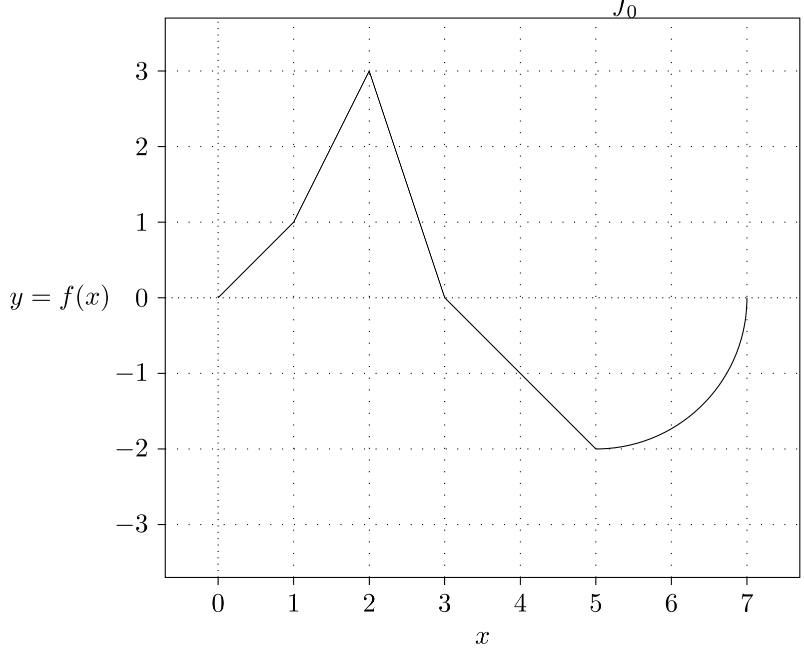
(Problem 239) Approximate $\displaystyle\int_1^3 x\,4^{-x}\,dx$ using a left Riemann sum with $n = 4$ rectangles of equal width.

(Problem 240) Approximate $\displaystyle\int_0^1 \cos^2(\pi x)\,dx$ using a Riemann sum with $n = 3$ rectangles.

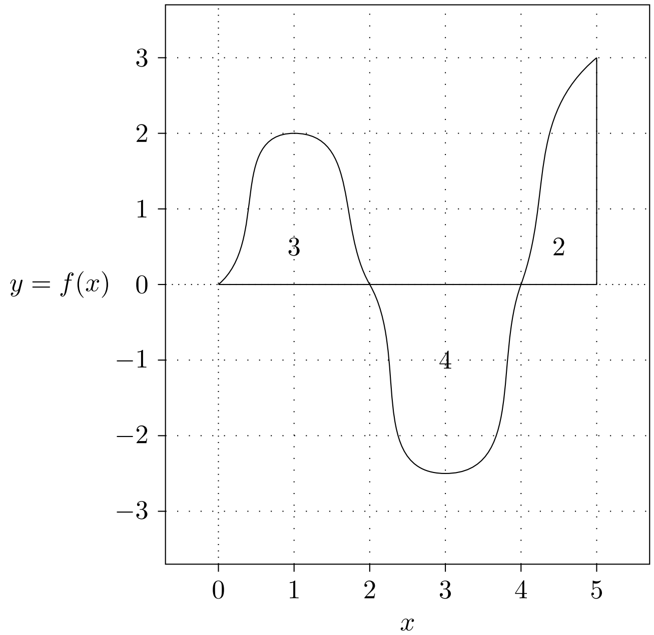
(Problem 241) Find $\displaystyle\int_{-4}^4 \sqrt{16-x^2}\,dx$ using geometry.

(Problem 242) Find $\displaystyle\int_2^4 \frac{x}{2}\,dx$ using geometry (not using antiderivatives).

(Problem 243) Here is the graph of $y = f(x)$. What is $\displaystyle\int_0^7 f(x)\,dx$?



(Problem 244) Here is the graph of $y = f(x)$. Some areas are indicated. What is $\int_0^5 f(x)\,dx$?



(Problem 245) Write a formula for the right Riemann sum for $\int_2^5 x^2\,dx$ with n rectangles. Your final answer may involve $\sum$.

(Problem 246) Find $\int_1^e \frac{3}{x}\,dx$.

(Problem 247) Find the total area between the curve $y = x^2 - 7x + 10$ and the x-axis over the interval $2 < x < 6$.

(Problem 248) Find $\int_2^3 x^3\,dx$.

(Problem 249) Find $\int_0^\pi \sin(3x)\,dx$.

(Problem 250) Find $\frac{d}{dx}\left(\int_2^x e^t\,dt\right)$.

(Problem 251) Find $\frac{d}{dx}\left(\int_x^5 \cos(t^2)\,dt\right)$.

(Problem 252) Find $\frac{d}{dx}\left(\int_3^{x^2} \frac{1}{1+t^2}\,dt\right)$.

(Problem 253) Find $\frac{d}{dx}\left(\int_{x^2}^{3x} t^5\,dt\right)$.

(Problem 254) Find $\int_{-3}^3 \sin(x^3)\,dx$.

(Problem 255) Find $\displaystyle\int_{-1}^{1} x^3 + x^5 + x^7 + x^9 \, dx$.

(Problem 256) Find $\displaystyle\int_{-2}^{2} 4 + x^2 - 3|x| \, dx$.

(Problem 257) Find the average value of $f(x) = x^3$ on the interval $[2, 6]$.

(Problem 258) Find the average value of $f(x) = \cos x$ on the interval $[0, \pi]$.

(Problem 259) Let $f(x) = x^2$. Find a number c such that $1 < c < 3$ and such that $f(c) = \dfrac{1}{3-1}\displaystyle\int_1^3 f(x)\,dx$.

(Problem 260) Find $\displaystyle\int \cos(x^3)\,x^2\,dx$.

(Problem 261) Find $\displaystyle\int \sin(e^x)\,e^x\,dx$.

(Problem 262) Find $\displaystyle\int (1 + x^2)^7\,x\,dx$.

(Problem 263) Find $\displaystyle\int \frac{1}{x^2 + 6x + 9}\,dx$.

(Problem 264) Find $\displaystyle\int \frac{1}{x^2 + 6x + 10}\,dx$. *Hint*: Let $u = x + 3$.

(Problem 265) Find $\displaystyle\int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x}\,dx$.

(Problem 266) Find $\displaystyle\int \cot x\,dx$.

(Problem 267) Find $\displaystyle\int_0^{\pi/2} \sin^3 x\,\cos x\,dx$.

(Problem 268) Find $\displaystyle\int_2^4 (x^4 - 3)^5\,x^3\,dx$.

(Problem 269) Find $\displaystyle\int_3^4 \sqrt{25 - x^2}\,x\,dx$.

Answer key

(Answer 1)
- (a) $f(1) = 3$.
- (b) $\lim_{x\to 1} f(x) = 1$.
- (c) $f(2) = 2$.
- (d) $\lim_{x\to 2} f(x)$ does not exist.
- (e) $\lim_{x\to 2^+} f(x) = 2$.
- (f) $\lim_{x\to 2^-} f(x) = 1$.
- (g) $f(3) = 1$.
- (h) $\lim_{x\to 3} f(x) = 1$.

(Answer 2) $\lim_{x\to 0} f(x) \approx 0.69315$.

(Answer 3) $\lim_{x\to 0^+} f(x) = 1$; $\lim_{x\to 0^-} f(x) = -1$.

(Answer 4) $\lim_{x\to 3} \dfrac{x^3 - 9x^2 + 9x - 27}{x - 3} = -18$.

(Answer 13) Because $-1 \leq \sin(1/(x-3)) \leq 1$ for all $x \neq 0$, and $(x-3)^2 \geq 0$ for we have that

$$x^2 - (x-3)^2 \leq x^2 + (x-3)^2 \sin(1/(x-3)) \leq x^2 + (x-3)^2$$

for all $x \neq 0$. Because $\lim_{x\to 3} x^2 - (x-3)^2 = \lim_{x\to 3} x^2 + (x-3)^2 = 9$, we have by the squeeze theorem that $\lim_{x\to 3} x^2 + (x-3)^2 \sin(1/(x-3)) = 9$.

(Answer 20)
- (a) $\lim_{x\to a^+} f(x) = \infty$ for $a = 4$.
- (b) $\lim_{x\to a^+} f(x) = -\infty$ for $a = 2$ and $a = 3$.
- (c) $\lim_{x\to a^-} f(x) = \infty$ for $a = 2$.
- (d) $\lim_{x\to a^-} f(x) = -\infty$ for $a = 3$ and $a = 4$.
- (e) There are no numbers a such that $\lim_{x\to a} f(x) = \infty$.
- (f) $\lim_{x\to a^-} f(x) = -\infty$ for $a = 3$.

(Answer 41)
- (a) $\lim_{x\to 0} g(x) = 1$.
- (b) $\lim_{x\to 1} f(x) = 0$.
- (c) The limit does not exist.
- (d) $\lim_{x\to 0} g(f(x)) = 1$.

(Answer 44) Suppose that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that, if $a < x < a + \delta$, then $f(x)$ exists and $|f(x) - L| < \varepsilon$. Then we say that $\lim_{x\to a^+} f(x) = L$.

(Answer 45) Suppose that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that, if $a - \delta < x < a$, then $f(x)$ exists and $|f(x) - L| < \varepsilon$. Then we say that $\lim_{x\to a^-} f(x) = L$.

(Answer 46) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $0 < |x - a| < \delta$, then $f(x)$ exists and $f(x) > M$. Then we say that $\lim_{x\to a} f(x) = \infty$.

(Answer 47) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $a < x < a + \delta$, then $f(x)$ exists and $f(x) > M$. Then we say that $\lim_{x\to a^+} f(x) = \infty$.

(Answer 48) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $a - \delta < x < a$, then $f(x)$ exists and $f(x) > M$. Then we say that $\lim_{x\to a^-} f(x) = \infty$.

(Answer 49) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $0 < |x - a| < \delta$, then $f(x)$ exists and $f(x) < -M$. Then we say that $\lim_{x\to a} f(x) = -\infty$.

(Answer 50) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $a < x < a + \delta$, then $f(x)$ exists and $f(x) < -M$. Then we say that $\lim_{x\to a^+} f(x) = -\infty$.

(Answer 51) Suppose that for every $M > 0$ there exists a $\delta > 0$ such that, if $a - \delta < x < a$, then $f(x)$ exists and $f(x) < -M$. Then we say that $\lim_{x\to a^-} f(x) = -\infty$.

(Answer 52) Suppose that for every $\varepsilon > 0$ there exists a $N > 0$ such that, if $x > N$, then $f(x)$ exists and $|f(x) - L| < \varepsilon$. Then we say that $\lim_{x\to\infty} f(x) = L$.

(Answer 53) Suppose that for every $\varepsilon > 0$ there exists a $N > 0$ such that, if $x < -N$, then $f(x)$ exists and $|f(x) - L| < \varepsilon$. Then we say that $\lim_{x\to-\infty} f(x) = L$.

(Answer 54) Suppose that for every $M > 0$ there exists a $N > 0$ such that, if $x > N$, then $f(x)$ exists and $f(x) > M$. Then we say that $\lim_{x\to\infty} f(x) = \infty$.

(Answer 55) Suppose that for every $M > 0$ there exists a $N > 0$ such that, if $x > N$, then $f(x)$ exists and $f(x) < -M$. Then we say that $\lim_{x\to\infty} f(x) = -\infty$.

(Answer 56) Suppose that for every $M > 0$ there exists a $N > 0$ such that, if $x < -N$, then $f(x)$ exists and $f(x) > M$. Then we say that $\lim_{x\to-\infty} f(x) = \infty$.

(Answer 57) Suppose that for every $M > 0$ there exists a $N > 0$ such that, if $x < -N$, then $f(x)$ exists and $f(x) < -M$. Then we say that $\lim_{x\to-\infty} f(x) = -\infty$.

(Answer 58) We want to find a δ such that, if $|x - 3| < \delta$, then $|x^2 - 9| < 1$.
We know that if $-1 < x^2 - 9 < 1$, then $|x^2 - 9| < 1$.
Adding 9 to all sides, we see that if $8 < x^2 < 10$, then $|x^2 - 9| < 1$.
The function $f(x) = x^2$ is increasing when $x > 0$. Thus, if $\sqrt{8} < x < \sqrt{10}$, then $8 < x^2 < 10$.
Subtracting 3, we see that if $\sqrt{8} - 3 < x - 3 < \sqrt{10} - 3$, then $|x^2 - 9| < 1$.
If $|x - 3| < \min(\sqrt{10} - 3, 3 - \sqrt{8})$, then $\sqrt{8} - 3 < x - 3 < \sqrt{10} - 3$ and so $|x^2 - 9| < 1$.
Thus, $\boxed{\delta = \min(\sqrt{10} - 3, 3 - \sqrt{8})}$ is a solution.
(If you have a calculator, you can compute $\sqrt{10} - 3 = 0.16228$ and $3 - \sqrt{8} = 0.17157$, so $\delta = \sqrt{10} - 3 = 0.16228$.)

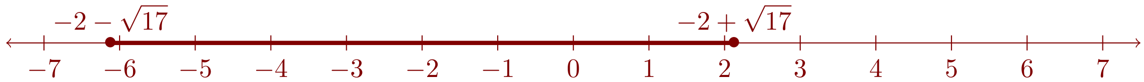
(Answer 59) We want to find a δ such that, if $|x-2|<\delta$, then $|x^2+4x-8|<5$.

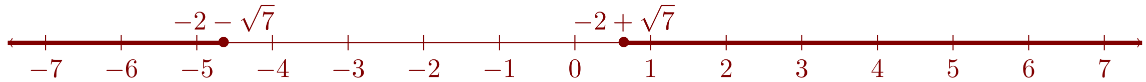
We know that if $-5<x^2+4x-8<5$, then $|x^2+4x-8|<5$.

Thus, we need both that $x^2+4x-13<0$ and $x^2+4x-3>0$.

Factoring (and using the quadratic formula), we see that we need $(x+2-\sqrt{13})(x+2+\sqrt{17})<0$ and $(x+2-\sqrt{11})(x+2+\sqrt{7})>0$.

Thus, we need $-2-\sqrt{17}<x<-2+\sqrt{17}$ and either $x>-2+\sqrt{7}$ or $x<-2-\sqrt{7}$.





Thus, we need either $-2-\sqrt{17}<x<-2-\sqrt{7}$ or $-2+\sqrt{7}<x<-2+\sqrt{17}$.

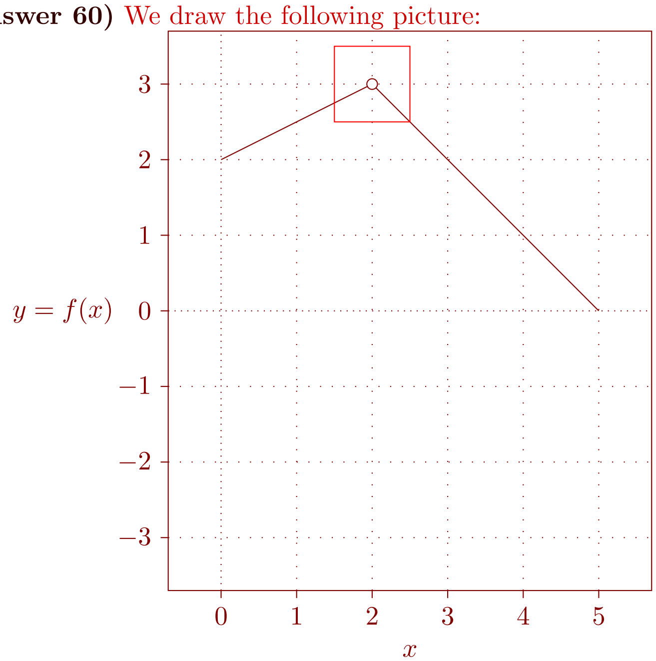
Notice that $2=-2+\sqrt{16}$, so $-2+\sqrt{7}<2<-2+\sqrt{17}$; thus, we care about the interval $-2+\sqrt{7}<x<-2+\sqrt{17}$.

Subtracting 2, we see that we need $-4+\sqrt{7}<x-2<-4+\sqrt{17}$.

Thus, if $|x-2|<\min(\sqrt{17}-4,4-\sqrt{7})$, then $-4+\sqrt{7}<x-2<-4+\sqrt{17}$ and so $|x^2+4x-8|<5$.

Thus, $\boxed{\delta=\min(\sqrt{17}-4,4-\sqrt{7})}$ is a solution.

(Answer 60) We draw the following picture:



Observe that the box has width 1; thus, $\boxed{\delta=1/2}$ is a solution.

(Answer 61) I choose $\varepsilon = 1/2$. Let $\delta > 0$. Let N be an integer larger than $\dfrac{1}{\pi\delta}$, so

$$\frac{1}{\pi\delta} < N.$$

Multiplying both sides of the inequality by the positive number δ/N, we see that

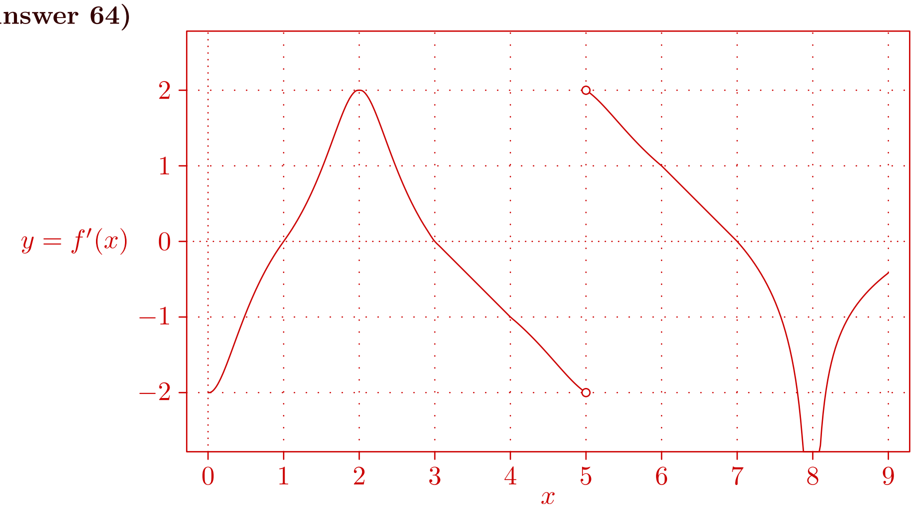
$$\frac{1}{N\pi} < \delta.$$

Let $x = 1/(N\pi)$. Then $0 < x < \delta$ and so $0 < |x - 0| < \delta$. We have that $f(x) = \cos(1/x) = \cos(N\pi) = \pm 1$. Then $|f(x) - 0| = |\pm 1| = 1 > 0.5 = \varepsilon$, and so 0 is not $\lim_{x\to 0} f(x)$.

(Answer 62) I choose $\varepsilon = \pi/2$. Let $\delta > 0$. Let x satisfy $-\delta < x < 0$. Then $0 < |x - 0| < \delta$. We have that $f(x) = \arctan(1/x)$, and $\arctan(z) < 0$ whenever $z < 0$. So $|f(x) - \pi/2| = \pi/2 - \arctan(1/x) \geq \pi/2 = \varepsilon$, and so $\pi/2$ is not $\lim_{x\to 0} f(x)$.

(Answer 63) I choose $\varepsilon = 1/2$. Let $\delta > 0$. Let x satisfy $0 < x < \delta$. Then $0 < |x - 0| < \delta$. Furthermore, $1/x > 0$, so $2^{1/x} > 1$ and $1 + 2^{1/x} > 2$ and $f(x) = \dfrac{1}{1 + 2^{1/x}} < \dfrac{1}{2}$. Thus, $f(x) - 1 < -1/2$ and so $|f(x) - 1| > 1/2 = \varepsilon$, and so 1 is not $\lim_{x\to 0} f(x)$.

(Answer 64)



(Answer 66) $\dfrac{d}{dx}(x^2)\Big|_{x=3} = \lim_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h} = \lim_{h\to 0} \dfrac{3^2 + 6h + h^2 - 3^2}{h} = \lim_{h\to 0} 6 + h = 6.$

(Answer 67) $y - 9 = 6(x - 3)$.

(Answer 80) $\dfrac{d}{dx}(3^x) = \lim_{h\to 0} \dfrac{3^{x+h} - 3^x}{h} = \lim_{h\to 0} \dfrac{3^x 3^h - 3^x}{h} = 3^x \lim_{h\to 0} \dfrac{3^h - 1}{h}.$

(Answer 81) If $x \leq 0$ then $\dfrac{d}{dx}(\log_3 x)$ does not exist, as $\log_3 x$ does not exist. Suppose $x > 0$. Then $\dfrac{d}{dx}(\log_3 x) = \lim_{h\to 0} \dfrac{\log_3(x+h) - \log_3 x}{h} = \lim_{h\to 0} \dfrac{\log_3 \frac{x+h}{x}}{h}$. We make the change of variables $h = qx$, so $q = h/x$. Since $x > 0$, we have that h is near but not equal to zero whenever q is near but not equal to zero. Thus, $\dfrac{d}{dx}(\log_3 x) = \lim_{h\to 0} \dfrac{\log_3 \frac{x+h}{x}}{h} = \lim_{q\to 0} \dfrac{\log_3 \frac{x+qx}{x}}{qx} = \dfrac{1}{x} \lim_{q\to 0} \dfrac{\log_3(1+q)}{q}.$

(Answer 144) $\frac{dy}{dx} = y\left(\frac{3}{x} + 3 - 5\tan x - \frac{36x}{x^2+1} - \frac{3e^x}{e^x+2}\right)$.

(Answer 172) Yes; $c = 4$.

(Answer 173) Yes; $c = 1$.

(Answer 174) No; f is not right continuous at 0 and therefore f is not continuous on $[0, 4]$.

(Answer 175) No; $f(x)$ is not differentiable at $x = 0$ and 0 is in $(-2, 3)$.

(Answer 176)
(a) Yes; $f(x)$ is continuous on $[-1, 1]$ and $f'(x) = 3x^2$ exists everywhere in $(-1, 1)$.
(b) By the mean value theorem, if the conditions holds, then the conclusion holds.

(Answer 177)
(a) No; $f(x)$ is not differentiable at $x = 0$.
(b) Yes; $f'(\frac{1}{\sqrt{27}}) = 1 = \frac{f(1) - f(-1)}{1 - (-1)}$.

(Answer 178)
(a) No; $f(x)$ is not differentiable at $x = 0$.
(b) No; $f'(x) = \frac{2}{3}x^{-1/3}$ is never zero, and $\frac{f(1) - f(-1)}{1 - (-1)} = 0$.

(Answer 185) $f(x)$ is decreasing on $(-\infty, 0]$ and increasing on $[0, \infty)$.