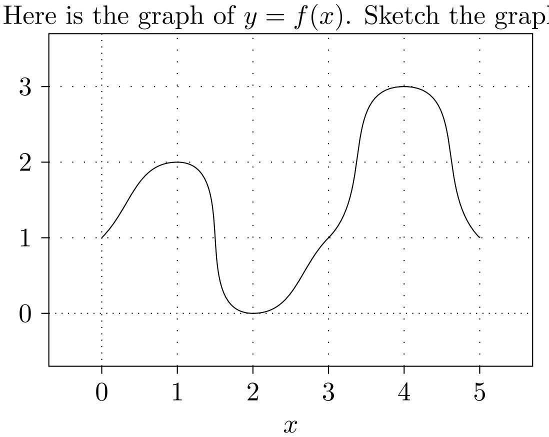
Math 2554H, Fall 2018

Exam 2 review

If you need it, the following information will be printed on the cover of the exam:
If f is a function, then $f'(x) = \dfrac{d}{dx}(f(x)) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.
$\lim\limits_{h\to 0} \dfrac{\sin h}{h} = 1$, $\lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} = 0$, $\sin(x+h) = \sin x \cos h + \sin h \cos x$, $\cos(x+h) = \cos x \cos h - \sin x \sin h$.

(Problem 1) Here is the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$.



(Problem 2) Use the limit definition of derivative to find $f'(1)$, where $f(x) = 3x + 2$.

(Problem 3) Use the limit definition of derivative to find $\dfrac{d}{dx}(x^2)\Big|_{x=3}$.

(Problem 4) Find the tangent line to the graph of $y = x^2$ at the point $(3, 9)$.

(Problem 5) Use the limit definition of derivative to find $g'(2)$, where $g(x) = x^3$.

(Problem 6) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt{x})\Big|_{x=25}$.

(Problem 7) Find the tangent line to the graph of $y = \sqrt{x}$ at the point $(25, 5)$.

(Problem 8) Use the limit definition of derivative to find $h'(2)$, where $h(x) = \dfrac{1}{x}$.

(Problem 9) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt[3]{x})\Big|_{x=0}$.

(Problem 10) Use the limit definition of derivative to find $f'(x)$, where $f(x) = 3x + 2$.

(Problem 11) Use the limit definition of derivative to find $\dfrac{d}{dx}(x^2)$.

(Problem 12) Use the limit definition of derivative to find $g'(x)$, where $g(x) = x^3$.

(Problem 13) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sqrt{x})$.

(Problem 14) Use the limit definition of derivative to find $h'(x)$, where $h(x) = \dfrac{1}{x^2}$.

(Problem 15) Use the limit definition of derivative to find $\dfrac{d}{dx}(\sin x)$.

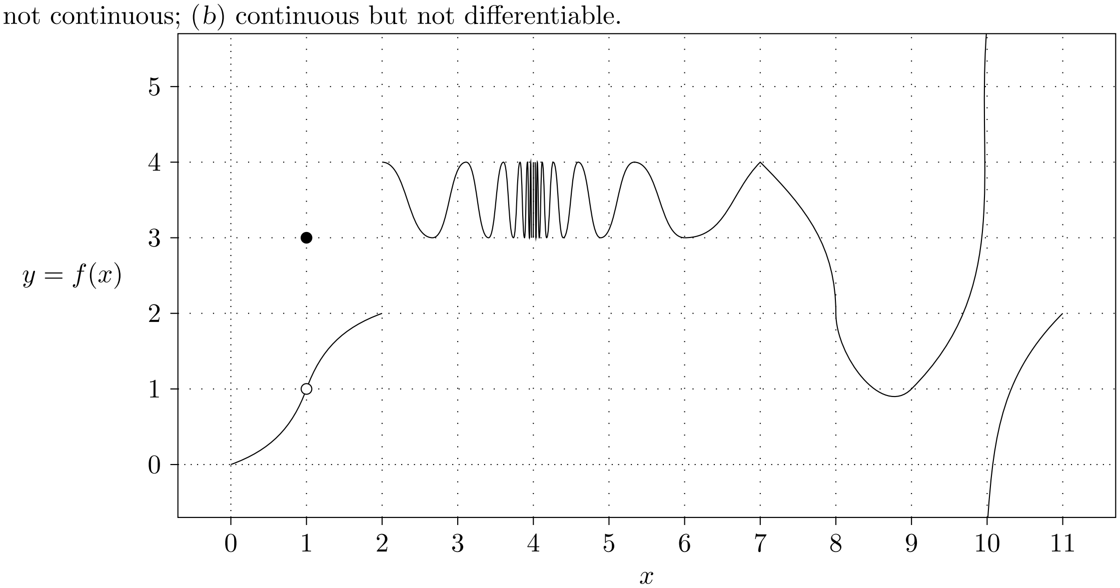
(Problem 16) Use the limit definition of derivative to find $\dfrac{d}{dx}(\cos x)$.

(Problem 17) Use the limit definition of derivative to find $\dfrac{d}{dx}(3^x)$. Your final answer may involve a limit as long as that limit does not depend on x.

(Problem 18) Use the limit definition of derivative to find $\dfrac{d}{dx}(\log_3 x)$. Your final answer may involve a limit as long as that limit does not depend on x.

(Problem 19) Let $f(x) = \dfrac{x^3 - 2x^2}{x - 2}$. What is $f'(2)$?

(Problem 20) Here is the graph of $y = f(x)$. Find all points x with $0 < x < 11$ at which the graph is (a) not continuous; (b) continuous but not differentiable.



(Problem 21) Let $f(x) = x^7 + 9x^3 + e^x$. Find $f'(x)$.

(Problem 22) Find $\dfrac{d}{dx}(3x^3 - 9e^x)$.

(Problem 23) Find $\dfrac{d}{dx}(3\sin x + 4\cos x)$.

(Problem 24) Find $\dfrac{d}{dx}(2\tan x - 3\csc x)$.

(Problem 25) Let $g(x) = \dfrac{3}{x^7} - 2\sqrt[5]{x}$. Find $g'(x)$, $g''(x)$, and $g^{(3)}(x)$.

(Problem 26) Find $\dfrac{d}{dx}(x^4 e^x)$.

(Problem 27) Find $\dfrac{d}{dx}(x^4 \cos x)$.

(Problem 28) Find $\dfrac{d}{dx}(x^5 e^x \cot x)$.

(Problem 29) Find $\dfrac{d}{dx}\left(\dfrac{x^5}{e^x+1}\right)$.

(Problem 30) Find $\dfrac{d}{dx}\left(\dfrac{e^x}{x^2+1}\right)$.

(Problem 31) Find $\dfrac{d}{dx}\left(\dfrac{x^2 \sin x}{2e^x+x}\right)$.

(Problem 32) Find $\dfrac{d^2}{dx^2}(\sec x)$.

(Problem 33) Find $f''(x)$, where $f(x) = \csc x$.

(Problem 34) Use the quotient rule to find $\dfrac{d}{dx}(\tan x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 35) Use the quotient rule to find $\dfrac{d}{dx}(\cot x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 36) Use the quotient rule to find $\dfrac{d}{dx}(\sec x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 37) Use the quotient rule to find $\dfrac{d}{dx}(\csc x)$. (You may use the fact that $\dfrac{d}{dx}(\sin x) = \cos x$ and $\dfrac{d}{dx}(\cos x) = -\sin x$.)

(Problem 38) Find $\displaystyle\lim_{x\to 0} \frac{\cos 3x - 1}{x}$.

(Problem 39) Find $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{x}$.

(Problem 40) Find $\displaystyle\lim_{x\to 0} \frac{\tan 5x}{x}$.

(Problem 41) Find $\displaystyle\lim_{x\to 0} \frac{\sec 2x - 1}{x}$.

(Problem 42) Find $\displaystyle\lim_{x\to 0} \frac{\cos^2 x - 1}{x}$.

(Problem 43) Find $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{x}$.

(Problem 44) Find $\lim_{x\to 2} \dfrac{\sin(x-2)}{4-x^2}$.

(Problem 45) Find $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$. *Hint*: Multiply by $\dfrac{1+\cos x}{1+\cos x}$.

(Problem 46) Find $\lim_{x\to 0} \dfrac{\tan 3x}{\sin 2x}$.

(Problem 47)

(*a*) Use the squeeze theorem to find $\lim_{x\to\pi/2^-} \dfrac{\cos x}{x-\pi/2}$.

(*b*) Use the squeeze theorem to find $\lim_{x\to\pi/2^-} \dfrac{1-\sin x}{x-\pi/2}$.

(Problem 49)

(*a*) Use the squeeze theorem to find $\lim_{x\to\pi^+} \dfrac{\sin x}{x-\pi}$.

(*b*) Use the squeeze theorem to find $\lim_{x\to\pi^+} \dfrac{\cos x+1}{x-\pi}$.

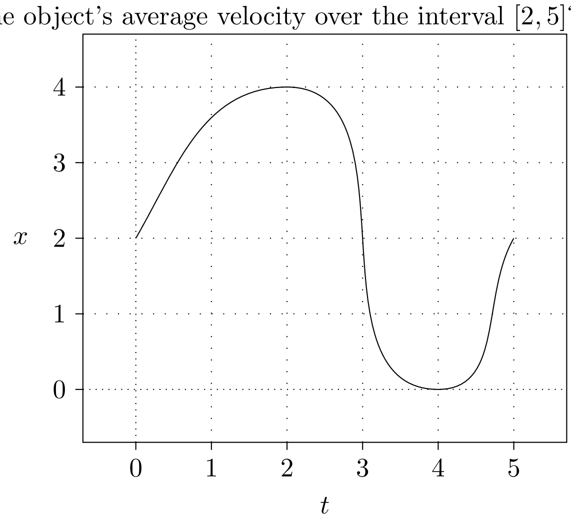
(Problem 51)

(*a*) Use the squeeze theorem to find $\lim_{x\to 3\pi/2^-} \dfrac{\cos x}{x-3\pi/2}$.

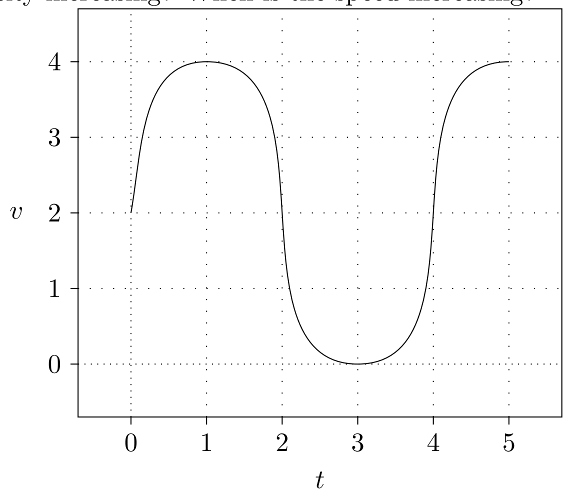
(*b*) Use the squeeze theorem to find $\lim_{x\to 3\pi/2^-} \dfrac{1+\sin x}{x-3\pi/2}$.

(Problem 53) Suppose that after t seconds, a thrown projectile's height is $h(t) = 4 + 19t - 5t^2$ meters. What is its average velocity over the interval $[1, 3]$? What is the object's maximum height? What is the object's instantaneous velocity at the moment that it hits the ground?

(Problem 54) The following graph shows the displacement of an object to the right of a fixed basepoint as a function of time. When is the object stationary? When is it moving left? When is it moving right? What is the object's average velocity over the interval $[2, 5]$?



(Problem 55) The following graph shows the velocity of an object as a function of time. When is the velocity increasing? When is the speed increasing?



(Problem 56) In 2015, the birth rate in a certain city was 5000 births/year and the death rate was 250 deaths/month. Neglecting immigration and emigration, what was the rate of change of population of the city? Be sure your answer includes units.

(Problem 57) Find $\dfrac{d}{dx}(\csc(7x))$.

(Problem 58) Find $\dfrac{d}{dx}(e^{3x})$.

(Problem 59) Find $\dfrac{d}{dx}(\tan(2x+5))$.

(Problem 60) Find $\dfrac{d}{dx}(e^{4x-3})$.

(Problem 61) Find $\dfrac{d}{dx}(\sin(x^3))$.

(Problem 62) Find $\dfrac{d^2}{dx^2}(\cos(x^4))$.

(Problem 63) Find $\dfrac{d}{dx}((\tan x)^4)$.

(Problem 64) Find $\dfrac{d}{dx}(\sqrt{x^3+5})$.

(Problem 65) Find $\dfrac{d}{dx}(\sqrt[3]{2x-4})$.

(Problem 66) Find $\dfrac{d^2}{dx^2}(\tan x)$.

(Problem 67) Find $f''(x)$, where $f(x) = \cot x$.

(Problem 68) Find $\frac{d}{dx}(e^{-x^2})$.

(Problem 69) Find $\frac{d}{dx}(\sec(e^x))$.

(Problem 70) Find $\frac{d}{dx}(\sin(e^{x^3}))$.

(Problem 71) Use the following table to find $\frac{d}{dx}(f(g(x)))\big|_{x=4}$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	25	6	7
2	3	3	4	2
3	2	5	3	16
4	5	15	1	8
5	1	9	5	4
6	6	27	2	14

(Problem 72) Use the following tables to find $\frac{dy}{dx}\big|_{x=2}$.

x	u	$\frac{du}{dx}$
1	2	8
2	3	14
3	1	2
4	4	7
5	6	16
6	5	4

u	y	$\frac{dy}{du}$
1	5	15
2	1	3
3	3	5
4	4	9
5	6	27
6	2	25

(Problem 73) Suppose that y satisfies $y^5 + x^4y^3 = x^7$. Find $\frac{dy}{dx}$.

(Problem 74) Suppose that y satisfies $\sin(x) + \sin(y) = y$. Find $\frac{d^2y}{dx^2}$.

(Problem 75) Find the tangent line to the hyperbola $x^2 - y^2 = 9$ at the point $(5, 4)$.

(Problem 76) Find $\frac{d}{dx}(\ln x)$ using implicit differentiation. Simplify your answer as much as possible.

(Problem 77) Find $\frac{d}{dx}(\log_5|x|)$.

(Problem 78) Find $\frac{d}{dx}(\ln(x^2+1))$.

(Problem 79) Find $\frac{d}{dx}(9^x)$.

(Problem 80) Find $\frac{d}{dx}(x^{\cos x})$.

(Problem 81) Let $y = \frac{x^3e^{3x}\cos^5 x}{(x^2+1)^{18}(e^x+2)^3}$. Find $\frac{dy}{dx}$. If it is notationally simpler to express your answer using y as well as x, do so.

(Problem 82) Find $\dfrac{d}{dx}(\arcsin x)$ using implicit differentiation. Simplify your answer as much as possible. For what x-values is your formula valid?

(Problem 83) Find $\dfrac{d}{dx}(\arccos x)$ using implicit differentiation. Simplify your answer as much as possible. For what x-values is your formula valid?

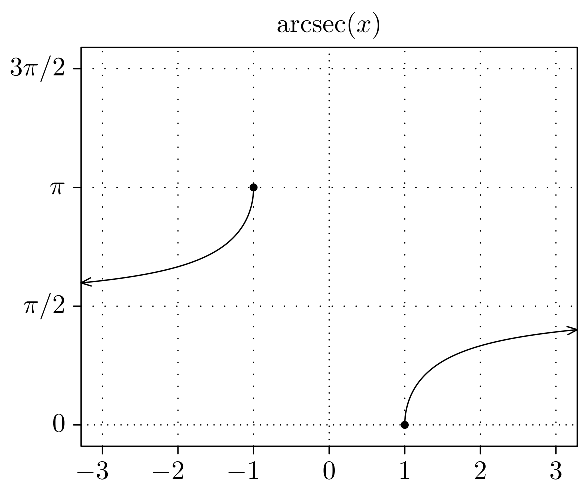
(Problem 84) Find $\dfrac{d}{dx}(\arctan x)$ using implicit differentiation. Simplify your answer as much as possible.

(Problem 85) Find $\dfrac{d}{dx}(\operatorname{arccot} x)$ using implicit differentiation. Simplify your answer as much as possible.

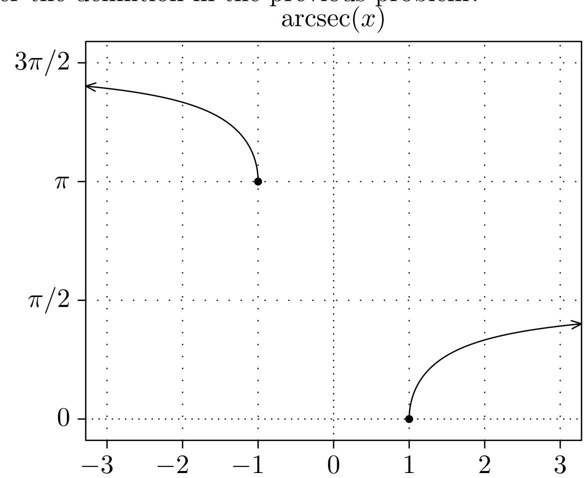
(Problem 86) Find $\dfrac{d}{dx}(\operatorname{arcsec} x)$, $x \geq 1$, using implicit differentiation. Simplify your answer as much as possible.

(Problem 87) Find $\dfrac{d}{dx}(\operatorname{arccsc} x)$, $x \geq 1$, using implicit differentiation. Simplify your answer as much as possible.

(Problem 88) I define $\operatorname{arcsec} x$ to be given by the following graph. If $x \leq 1$, is $\dfrac{d}{dx}(\operatorname{arcsec} x)$ positive or negative?



(Problem 89) I define $\operatorname{arcsec} x$ to be given by the following graph. If $x \leq 1$, is $\dfrac{d}{dx}(\operatorname{arcsec} x)$ positive or negative? Why might I prefer the definition in the previous problem?



(Problem 90) Let $f(x) = x^3 + 3x^5 + 2$. Notice that $f(1) = 6$. Find $\dfrac{d}{dx}(f^{-1}(x))\Big|_{x=6}$.

(Problem 91) Here is a table showing some values of x, $f(x)$, and $f'(x)$. Find $(f^{-1})'(3)$.

x	$f(x)$	$f'(x)$
1	−1	1
2	0	4
3	2	0
4	3	2
5	4	5
6	7	8

Answer key

(Answer 3) $\frac{d}{dx}(x^2)\big|_{x=3} = \lim_{h\to 0} \frac{(3+h)^2 - 3^2}{h} = \lim_{h\to 0} \frac{3^2 + 6h + h^2 - 3^2}{h} = \lim_{h\to 0} 6 + h = 6.$

(Answer 4) $y - 9 = 6(x-3)$.

(Answer 17) $\frac{d}{dx}(3^x) = \lim_{h\to 0} \frac{3^{x+h} - 3^x}{h} = \lim_{h\to 0} \frac{3^x 3^h - 3^x}{h} = 3^x \lim_{h\to 0} \frac{3^h - 1}{h}.$

(Answer 18) If $x \leq 0$ then $\frac{d}{dx}(\log_3 x)$ does not exist, as $\log_3 x$ does not exist. Suppose $x > 0$. Then $\frac{d}{dx}(\log_3 x) = \lim_{h\to 0} \frac{\log_3(x+h) - \log_3 x}{h} = \lim_{h\to 0} \frac{\log_3 \frac{x+h}{x}}{h}$. We make the change of variables $h = qx$, so $q = h/x$. Since $x > 0$, we have that h is near but not equal to zero whenever q is near but not equal to zero. Thus, $\frac{d}{dx}(\log_3 x) = \lim_{h\to 0} \frac{\log_3 \frac{x+h}{x}}{h} = \lim_{q\to 0} \frac{\log_3 \frac{x+qx}{x}}{qx} = \frac{1}{x} \lim_{q\to 0} \frac{\log_3(1+q)}{q}.$

(Answer 81) $\frac{dy}{dx} = y\left(\frac{3}{x} + 3 - 5\tan x - \frac{36x}{x^2+1} - \frac{3e^x}{e^x+2}\right).$