Math 2574, Fall 2017

You will be allowed a single-sided 8.5" $\times$11" sheet of notes. You are responsible for writing down all formulas you might need.

(Problem 1) Find $\iint_R x^2y\,dA$, where R is the triangle with vertices $(1,2)$, $(1,5)$ and $(3,2)$.

(Problem 2) Find $\iint_R xy\,dA$, where R is bounded by the curves $y=2x$ and $x=y^2$.

(Problem 3) Find $\iint_R x^2y^2\,dA$, where R is the region in quadrants 1 and 4 bounded by the semicircle of radius 3 centered at $(0,0)$.

(Problem 4) Find $\int_0^3\int_{2x}^6 e^{-y^2}\,dy\,dx$.

(Problem 5) Find $\iint_R x^2+xy+y^2\,dx\,dy$, where $R=\{(x,y):0<x<y, 1<x^2+y^2<4\}$.

(Problem 6) Find $\iint_R 3y^2\,dA$, where R is bounded by the curves $y=0$, $y=2x+4$, and $y=x^3$.

(Problem 7) Find the volume of the segment of the cylinder $x^2+y^2=25$ bounded above by the plane $z=x+y+10$ and below by $z=0$.

(Problem 8) Find $\iint_R x+y\,dA$, where $R=\{(x,y):x+y>0, 4<x^2+y^2<9\}$.

(Problem 9) Find $\iint_R x+3y\,dA$, where $R=\{(x,y):0<x<y, 1<x^2+y^2<4\}$.

(Problem 10) Find $\iint_R \frac{1}{\sqrt{x^2+y^2+1}}\,dA$, where R is the region in the first quadrant bounded by the semicircle of radius 2 centered at the origin.

(Problem 11) Find the area of the region bounded by the curve $r=\sin(3\theta)$.

(Problem 12) Find the volume of the solid bounded by the paraboloid $z=15-3x^2-3y^2$ and the plane $z=3$.

(Problem 13) Find $\int_2^3\int_4^7\int_0^1 xy^2\sin(\pi z)\,dx\,dy\,dz$.

(Problem 14) Find the volume of the wedge bounded by the parabolic cylinder $y=x^2$ and the planes $z=y-4x+6$ and $z=4x-y$.

(Problem 15) Find $\iiint_D ze^{-x^2-y^2}\,dV$, where D is the region $\{(x,y,z):x^2+y^2<1,\ 2<z<3\}$.

(Problem 16) Find $\iiint_D z\,dx\,dy\,dz$, where D is the region $\{(x,y,z):z^2>x^2+y^2+1,\ 0<z<10\}$.

(Problem 17) Find the volume of the region outside the cone $\{(x,y,z):z>\frac{3}{4}\sqrt{x^2+y^2}\}$, inside the sphere $\{(x,y,z):x^2+y^2+z^2<25\}$, and above the plane $z=0$.

(Problem 18) Find $\iiint_D z^2\,dV$, where D is the region $\{(x,y,z):4<x^2+y^2+z^2<9\}$.

(Problem 19) Describe the region $\{(\rho,\theta,\varphi):0<\rho<6\cos\varphi, 0\le\theta<2\pi, 0\le\varphi\le\pi\}$ using Cartesian coordinates.

(Problem 20) Find the center of mass of the thin rod of length 5 with density function $\rho(x)=2+x+x^3$, $0\le x\le 5$.

(Problem 21) Find the centroid of the region in the xy-plane bounded by the curves $y=x$ and $y=x^4$.

(Problem 22) Find the center of mass of the truncated half-cone $\{(x, y, z) : x > 0, \sqrt{x^2 + y^2} < z < 3\}$. (Assume this solid has constant density.)

(Problem 23) Let $S = \{(u, v) : 0 < u < 1, 0 < v < 1\}$. Find the image of S in the xy-plane under the transformation $x = u + 2v$, $y = v + 2u$.

(Problem 24) Let $S = \{(u, v) : 0 < u < 1, 0 < v < 1\}$. Find the image of S in the xy-plane under the transformation $x = u^2 - v^2$, $y = 2uv$.

(Problem 25) Find $\iint_R xy\,dx\,dy$, where R is the parallelogram with vertices at $(3, 2)$, $(5, 4)$, $(4, -1)$, and $(6, 1)$.

(Problem 26) Find $\iint_R \left(\frac{x-y}{x+y}\right)^6 dx\,dy$, where R is the parallelogram with vertices at $(1, 1)$, $(3, 3)$, $(0, 2)$, and $(2, 4)$.

(Problem 27) Find $\int_0^{\pi/2} \int_0^{1/(\cos\theta+\sin\theta)} r^6 \cos^5\theta\,dr\,d\theta$.

(Problem 28) Find $\iiint_D z\,dV$, where D is bounded by the cone $(z+3)^2 = 4(x-2)^2+9y^2$ and the xy-plane.

(Problem 29) Find the area of the region in the first quadrant bounded by the curves $y = x^3+1$, $y = x^3+2$, $y = 3 - x^3$ and $y = 4 - x^3$.