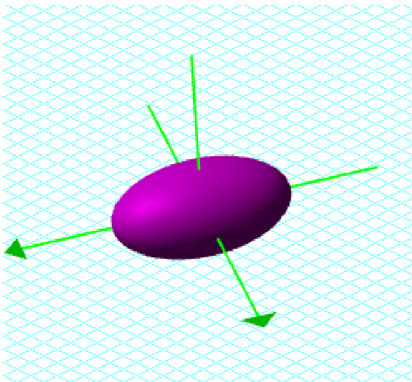
Math 2574, Fall 2017

You will be allowed a single-sided 8.5"×11" sheet of notes. You are responsible for writing down all formulas you might need.

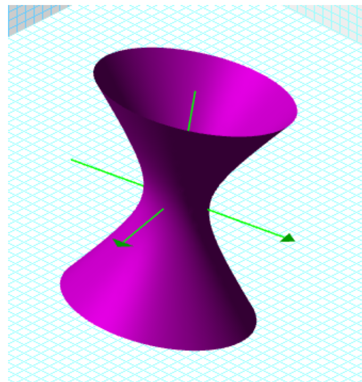
(Problem 1) Consider the surface $x^2 + 2y^2 + 4z^2 = 9$.
(a) Find the equations for the xy-trace, the xz-trace, and the yz-trace.
(b) Plot the xy-trace, the xz-trace, and the yz-trace.
(c) Classify the surface.

(Problem 2) Classify the following surfaces.

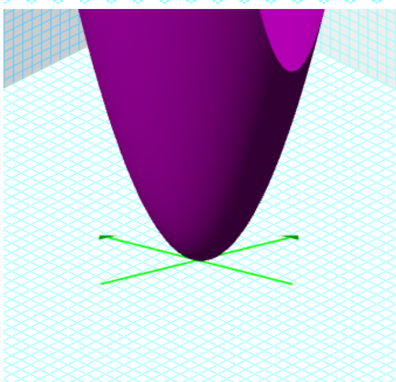
(a)



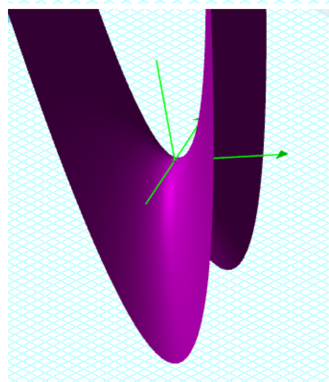
(b)



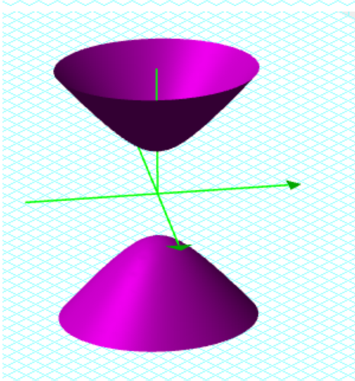
(c)



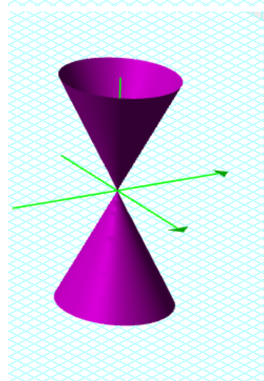
(d)



(e)



(f)



(Problem 3) Find the domain of the function $f(x, y) = \ln(x^2 + y^2)$.

(Problem 4) Find the domain of the function $f(x, y) = \ln(x^2 y^2)$.

(Problem 5) What is the range of the function $f(x, y) = e^{x+y}$?

(Problem 6) Sketch a graph of $f(x, y) = e^{-x^2-y^2}$.

(Problem 7) Sketch a graph of $f(x, y) = \sin(xy)$.

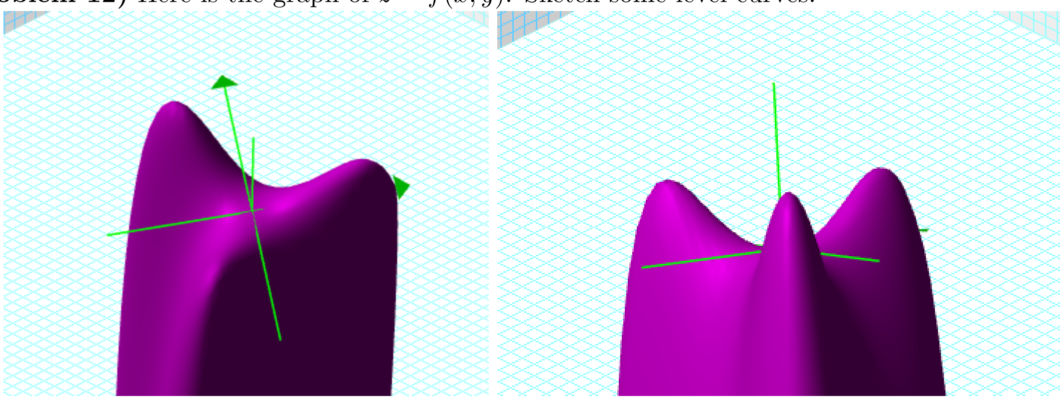
(Problem 8) Sketch a graph of $f(x, y) = \frac{1}{x-y}$.

(Problem 9) Sketch some representative level curves of the function $z = x - 2y + 1$.

(Problem 10) Sketch some representative level curves of the function $z = \frac{x^2}{9} + y^2$.

(Problem 11) Sketch some representative level curves of the function $z = \frac{x^2}{9} - y^2$.

(Problem 12) Here is the graph of $z = f(x, y)$. Sketch some level curves.



(Problem 13) Evaluate $\lim_{(x,y)\to(3,-3)} \frac{x^2+y^2}{x+y}$.

(Problem 14) Evaluate $\lim_{(x,y)\to(2,1)} \frac{x-2y-1}{1+\sqrt{x-2y}}$.

(Problem 15) Show that $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$ does not exist.

(Problem 16) Show that $\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$ does not exist.

(Problem 17) Show that $\lim_{(x,y)\to(3,1)} \frac{x+2y-5}{x-2y-1}$ does not exist.

(Problem 18) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^6+y^3}$ does not exist.

(Problem 19) Where is the function $f(x, y) = \ln(x^2y^2)$ continuous?

(Problem 20) Where is the function $f(x, y) = \sqrt{x - 2y}$ continuous?

(Problem 21) Let $f(x, y, z) = x^2 + yz$. Use the limit definition to find $D_{\langle 3/13,4/13,12/13\rangle} f(1, 2, 3)$.

(Problem 22) Let $f(x, y, z) = x^2 + \sin(yz) + xe^z$. Find $\nabla f(x, y, z)$.

(Problem 23) Let $z = \cos(xy^2)$, $x = t^3$, $y = t$. Find $\frac{dz}{dt}$.

(Problem 24) Use a tree diagram to write the Chain Rule formula for $\frac{ds}{dt}$ if s is a function of p and q, p is a function of r and t, and r and q are functions of t.

(Problem 25) The function $z = f(x, y)$ is given implicitly by the formula $z^5 - yz + x = 1$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(Problem 26) Let $f(x, y) = x^3 + 3y^2 - 2$.

(a) Find a unit vector in the direction of the steepest ascent of f at the point $(x, y) = (2, 1)$.

(b) Find a unit vector in the direction of the steepest descent of f at the point $(x, y) = (2, 1)$.

(c) Find a unit vector in the direction of no change in f at the point $(x, y) = (2, 1)$.

(d) Consider the level curve $x^3 + 3y^2 - 2 = 9$. Find a (parametric or nonparametric) equation for the tangent line to this curve at the point $(2, 1)$.

(Problem 27) Find the equation of the plane tangent to the surface $z = x^2 - y^2 + xy$ at the point $(2, 3, 1)$.

(Problem 28) Find the equation of the plane tangent to the surface $x^2 + 2y^2 + 3z^2 = 20$ at the point $(3, 2, 1)$.

(Problem 29) Find a linear approximation to the function $f(x, y) = \ln(x^2 + y^2)$ at the point $(x, y) = (0.8, -0.6)$. Then use it to estimate $f(0.81, -0.58)$.

(Problem 30) Find the critical points of the function $f(x, y) = x^2 + xy + y^2 + x$. Use the second derivative test to classify these critical points or show that the test is inconclusive.

(Problem 31) Find the critical points of the function $f(x, y) = x^2 + 4xy + y^2 + y$. Use the second derivative test to classify these critical points or show that the test is inconclusive.

(Problem 32) Find the critical points of the function $f(x, y) = xy^2 - 2xy + x$. Use the second derivative test to classify these critical points or show that the test is inconclusive.

(Problem 33) Use Lagrange multipliers to find the absolute maximum and absolute minimum of $f(x, y) = 3x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

(Problem 34) Find the point on the cone $z^2 = 3x^2 + 4y^2$ that is closest to the point $(3, 2, 1)$.

(Problem 35) Find the maximum and minimum values of $f(x, y) = x^2 + y^2 - x$ on the region $\{(x, y) : (x - 3)^2 + (y - 2)^2 \leq 16\}$.

(Problem 36) Find the maximum and minimum values of $f(x, y, z) = 3x + 2y + 1$ subject to the two constraints $x^2 + 4y^2 + 9z^2 = 1$ and $x + y + z = 0$.

(Problem 37) Let $R = \{(x, y) : 3 < x < 6, 0 < y < 2\}$. Find $\int\int_R x^2 + y^3 \, dA$.

(Problem 38) Find the volume of the solid $\{(x, y, z) : 2 \leq x \leq 3,\ 1 \leq y \leq 2,\ 0 \leq z \leq 2 + \sin(x)\cos(y)\}$.

Answer key

(Answer 2)

(a) An ellipsoid.
(b) A hyperboloid of one sheet.
(c) An elliptic paraboloid.
(d) A hyperbolic paraboloid.
(e) A hyperboloid of two sheets.
(f) An elliptic cone.

(Answer 3) The domain of the function $f(x, y) = \ln(x^2 + y^2)$ is $\{(x, y) : x \neq 0 \text{ or } y \neq 0\}$.

(Answer 4) The domain of the function $f(x, y) = \ln(x^2 y^2)$ is $\{(x, y) : x \neq 0 \text{ and } y \neq 0\}$.