

(AB 1) Let $\gamma_k(t) = t e^{i\theta_k}$ for $0 \leq t \leq 1$, where $\theta_0 = 0$, $\theta_1 = \pi/8$, $\theta_2 = \pi/4$, $\theta_3 = 3\pi/8$, and $\theta_4 = \pi/2$.

(a) On one set of axes, sketch γ_0 , γ_1 , γ_2 , γ_3 , and γ_4 .

(b) On a second set of axes, sketch $f \circ \gamma_k$ for $k = 0, 1, 2, 3, 4$, where $f(z) = z^4$.

(AB 2) Let $\gamma_k(t) = r_k e^{it}$ for $0 \leq t \leq \theta_k$, where $r_0 = 1/2$, $r_1 = 1$, $r_2 = 3/2$, $r_3 = 2$, and where $\theta_0 = \theta_1 = \pi/2$, $\theta_2 = \pi/4$, and $\theta_3 = \pi$.

(a) On one set of axes, sketch γ_0 , γ_1 , γ_2 , and γ_3 .

(b) On a second set of axes, sketch $f \circ \gamma_k$ for $k = 0, 1, 2, 3$, where $f(z) = z^2$.

On the previous problems, it may help to have polar graph paper; you can generate polar graph paper at <https://incompetech.com/graphpaper/polar/> or draw your own in LaTeX <http://www.texample.net/tikz/examples/polar-coordinates-template/> <http://tex.stackexchange.com/questions/169624/creating-a-polar-grid-with-tikz>

(AB 3) Prove that $\sin^2 z + \cos^2 z = 1$ for all $z \in \mathbb{C}$ using (a) Theorem 3.6.1; (b) the relations $\cos z = \frac{e^{iz} + e^{-iz}}{2}$, $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$.

(AB 4) Let x and y be real numbers. Find formulas for $\sin(x + iy)$ and $\cos(x + iy)$. Your answer should be of the form $\sin(x + iy) = f(x, y) + i g(x, y)$, $\cos(x + iy) = h(x, y) + i k(x, y)$, for real-valued functions f , g , h and k .

(AB 5) Let U be the quarter circle $\{x + iy : x > 0, y > 0, x^2 + y^2 < 5\}$. Find a conformal mapping $f : U \mapsto \mathbb{D} = D(0, 1)$.

(AB 6) Let $U = \mathbb{C} \setminus \{x : x \in \mathbb{R}, x \geq 0\}$. Find a conformal mapping $f : U \mapsto \mathbb{D} = D(0, 1)$.