(AB 1) For each of the following initial-value problems, tell me whether Theorem 4.1.1 guarantees a unique solution to the initial-value problem. If not, tell me whether there are (i) infinitely many solutions to the initial-value problem, or (ii) no solutions to the initial-value problem.

(a) $\dfrac{d^2y}{dx^2} + \dfrac{1}{x-6}\dfrac{dy}{dx} + y = \dfrac{1}{x+5}$, $y(0) = -2$, $y'(0) = 4$.

(b) $\dfrac{d^2y}{dx^2} - x^3\dfrac{dy}{dx} - y = x^5$, $y(0) = 6$, $y'(0) = 7$, $y''(0) = 0$.

(c) $\dfrac{d^4y}{dx^4} + \dfrac{1}{x^2+1}\dfrac{d^2y}{dx^2} - \tan(x)y = \sin(x)$, $y(0) = 2$, $y'(0) = 7$, $y''(0) = 9$, $y'''(0) = 16$.

(d) $\dfrac{d^3y}{dx^3} + \dfrac{1}{x^2+1}\dfrac{d^2y}{dx^2} - \cos(x)y = \arctan x$, $y(0) = 6$, $y'(0) = 7$

(AB 2) For each of the following initial-value problems, tell me whether Theorem 4.1.1 guarantees a unique solution to the initial-value problem. If not, tell me whether there are (i) infinitely many solutions to the initial-value problem, or (ii) no solutions to the initial-value problem.

(a) $\dfrac{d^2y}{dx^2} + 25y = e^{-x}\sin(5x)$, $y(7) = 3$, $y'(7) = 2$, $y''(7) = 1$.

(b) $\dfrac{d^4y}{dx^4} + \dfrac{x}{1+x^2}\dfrac{d^2y}{dx^2} + \dfrac{1}{1+x^4}y = 0$, $y(3) = 1$, $y'(3) = 2$, $y''(3) = 3$.

(c) $\dfrac{d^3y}{dx^3} + \ln(1+x^2)\dfrac{d^2y}{dx^2} + x^5\dfrac{dy}{dx} + y = \arctan x$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -1$.

Answers to selected problems

(AB 1) For each of the following initial-value problems, tell me whether Theorem 4.1.1 guarantees a unique solution to the initial-value problem. If not, tell me whether there are (i) infinitely many solutions to the initial-value problem, or (ii) no solutions to the initial-value problem.

(a) $\dfrac{d^2y}{dx^2} + \dfrac{1}{x-6}\dfrac{dy}{dx} + y = \dfrac{1}{x+5}$, $y(0) = -2$, $y'(0) = 4$.

We are guaranteed a unique solution.

(b) $\dfrac{d^2y}{dx^2} - x^3\dfrac{dy}{dx} - y = x^5$, $y(0) = 6$, $y'(0) = 7$, $y''(0) = 0$.

No solution exists.

(c) $\dfrac{d^4y}{dx^4} + \dfrac{1}{x^2+1}\dfrac{d^2y}{dx^2} - \tan(x)y = \sin(x)$, $y(0) = 2$, $y'(0) = 7$, $y''(0) = 9$, $y'''(0) = 16$.

We are guaranteed a unique solution.

(d) $\dfrac{d^3y}{dx^3} + \dfrac{1}{x^2+1}\dfrac{d^2y}{dx^2} - \cos(x)y = \arctan x$, $y(0) = 6$, $y'(0) = 7$

Infinitely many solutions exist.