Math 2584, Fall 2023

Exam 2 will occur on Friday, October 13, 2023, at 12:55 p.m., in PHYS 133.

You are allowed a non-graphing calculator and a double-sided 3 inch by 5 inch card of notes.

Carefully review Problem 5 from Exam 1.

(AB 1) Find the general solution to the following differential equations.

(a) $\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 85x = 0.$

(b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0.$

(c) $\frac{d^4z}{dt^4} + 7\frac{d^2z}{dt^2} - 144z = 0.$

(d) $\frac{d^4w}{dt^4} - 8\frac{d^2w}{dt^2} + 16w = 0.$

(AB 2) Solve the following initial-value problems. Express your answers in terms of real functions.

(a) $9\frac{d^2v}{dt^2} + 6\frac{dv}{dt} + 2v = 0$, $v(0) = 3$, $v'(0) = 2$.

(b) $\frac{d^2u}{dt^2} + 10\frac{du}{dt} + 25u = 0$, $u(0) = 1$, $u'(0) = 4$.

(c) $8\frac{d^2f}{dt^2} - 6\frac{df}{dt} + f = 0$, $f(0) = 3$, $f'(0) = 1$.

(AB 3) A spring is suspended vertically. When an object with mass 5 kg is attached, it stretches the spring to a new equilibrium 4 cm lower. The system moves in a medium which damps it with damping constant 16 newton $\cdot$ seconds/meter. The object is pulled down an additional 2 cm and is released with initial velocity 3 meters/second upwards.

Write the differential equation and initial conditions that describe the position of the object. You may use 9.8 meters/second^2 for the acceleration of gravity.

(AB 4) A 2-kg object is attached to a spring with constant 80 N/m and to a viscous damper with damping constant β. The object is pulled down to 10cm below its equilibrium position and released with no initial velocity.

Write the differential equation and initial conditions that describe the position of the object.

If $\beta = 20\ \mathrm{N \cdot s/m}$, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

If $\beta = 30\ \mathrm{N \cdot s/m}$, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

(AB 5) A 3-kg object is attached to a spring with constant k and to a viscous damper with damping constant 42 N·sec/m. The object is set in motion from its equilibrium position with initial velocity 5 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object.

Find the values of k for which the system is underdamped, overdamped, and critically damped. Be sure to include units for k.

(AB 6) A 4-kg object is attached to a spring with constant 70 N/m and to a viscous damper with damping constant β. The object is pushed up to 5cm above its equilibrium position and released with initial velocity 3 m/s downwards.

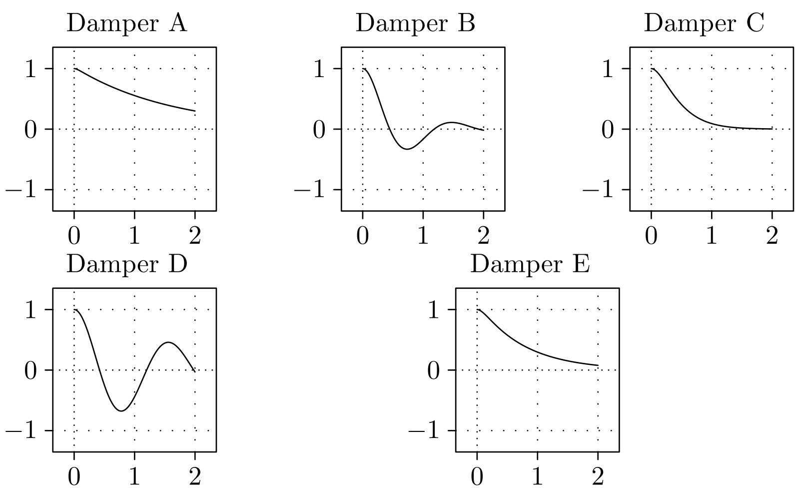
Write the differential equation and initial conditions that describe the position of the object. Then find the value of β for which the system is underdamped, overdamped, and critically damped. Be sure to include units for β.

(AB 7) An object of mass m is attached to a spring with constant 80 N/m and to a viscous damper with damping constant 20 N·s/m. The object is pulled down to 5cm below its equilibrium position and released with initial velocity 3 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object. Then find the value of m for which the system is underdamped, overdamped, or critically damped. Be sure to include units for m.

(AB 8) Five objects, each with mass 3 kg, are attached to five springs, each with constant 48 N/m. Five dampers with unknown constants are attached to the objects. In each case, the object is pulled down to a distance 1 cm below the equilibrium position and released from rest. You are given that the system is critically damped in exactly one of the five cases.

Here are the graphs of the objects' positions with respect to time:



(a) For which damper is the system critically damped?
(b) For which dampers is the system overdamped?
(c) For which dampers is the system underdamped?

(d) Which damper has the highest damping constant? Which damper has the lowest damping constant?

(AB 9) An object with mass 5 kg stretches a spring 4 cm. It is attached to a viscous damper with damping constant 16 newton · seconds/meter. The object is pulled down an additional 2 cm and is released with initial velocity 3 meters/second upwards.

Using only first derivatives, write the differential equations and initial conditions for the object's position. That is, write a first-order system involving the object's position. You may use 9.8 meters/second^2 for the acceleration of gravity.

(AB 10) Imperial stormtroopers and Rebel Alliance fighters battle each other on an open plain, where both groups can easily see and aim at all members of the other group. Every minute, each stormtrooper has a 2% chance of killing a rebel, and each rebel has a 5% chance of killing a stormtrooper. There are initially 4000 stormtroopers and 1000 rebels.

Write the initial value problem for the number of stormtroopers and rebels still alive.

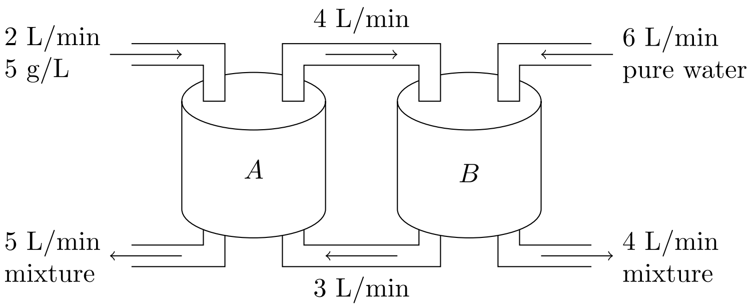
(AB 11) Jedi knights and Sith lords battle in a dense forest. Every minute, each Jedi has a 1% chance of finding each Sith lord. If a Jedi finds a Sith lord, they fight; the Jedi has a 60% chance of dying and the Sith lord has a 40% chance of dying. Initially there are 90 Jedi and 50 Sith lords.

Write the initial value problem for the number of Jedi and Sith lords still alive.

(AB 12) A forest is inhabited by rabbits and foxes. Every year, each rabbit gives birth to 3 babies and each fox gives birth to 5 babies on average. Foxes try to catch and eat rabbits; each month, each fox has a 0.02% chance of catching each of the rabbits present in the forest. Each month, every fox that does not eat a rabbit starves to death.

Write the differential equations for the number of foxes and rabbits in the forest.

(AB 13) Consider the following system of tanks. Tank A initially contains 200 L of water in which 3 kg of salt have been dissolved, and Tank B initially contains 300 L of water in which 2 kg of salt have been dissolved. Salt water flows into each tank at the rates shown, and the well-stirred solution flows between the two tanks and is drained away through the pipes shown at the indicated rates.



Write the differential equations and initial conditions that describe the amount of salt in each tank.

(AB 14) Write the system

$$\frac{dx}{dt} = 6x + 8y, \qquad \frac{dy}{dt} = -2x - 2y$$

in matrix form.

(AB 15) Write the system

$$\frac{dx}{dt} = 5y, \qquad \frac{dy}{dt} = 4y - x$$

in matrix form.

(AB 16) Find the solution to the initial-value problem

$$\frac{dx}{dt} = 6x + 8y, \qquad \frac{dy}{dt} = -2x - 2y, \qquad x(0) = -5, \quad y(0) = 3.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(AB 17) Find the solution to the initial-value problem

$$\frac{dx}{dt} = 7x - \frac{9}{2}y, \qquad \frac{dy}{dt} = -18x - 17y, \qquad x(0) = 0, \quad y(0) = 1.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(AB 18) Find the solution to the initial-value problem

$$\frac{dx}{dt} = -6x + 4y, \qquad \frac{dy}{dt} = -\frac{1}{4}x - 4y, \qquad x(0) = 1, \quad y(0) = 4.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(AB 19) Find the solution to the initial-value problem

$$\frac{dx}{dt} = -2x + 2y, \qquad \frac{dy}{dt} = 2x - 5y, \qquad x(0) = 1, \quad y(0) = 0.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(AB 20) You are given that the general solution to the system

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 11 & -4 \\ 9 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

may be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 e^{5t} \begin{pmatrix} 2 \\ 3 \end{pmatrix} + C_2 e^{5t} \begin{pmatrix} 2t+1 \\ 3t+1 \end{pmatrix}.$$

Find the solution to the initial value problem

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 11 & -4 \\ 9 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$

Answer key

(AB 1) Find the general solution to the following differential equations.

(a) $\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 85x = 0$.

If $\frac{d^2x}{dt^2}+12\frac{dx}{dt}+85x = 0$, then $x = C_1e^{-6t}\cos(7t)+C_2e^{-6t}\sin(7t)$.

(b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0$.

If $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = 0$, then $y = C_1e^{(-2+\sqrt{2})t} + C_2e^{(-2-\sqrt{2})t}$.

(c) $\frac{d^4z}{dt^4} + 7\frac{d^2z}{dt^2} - 144z = 0$.

If $\frac{d^4z}{dt^4} + 7\frac{d^2z}{dt^2} - 144z = 0$, then $z = C_1e^{3t} + C_2e^{-3t} + C_3\cos 4t + C_4\sin 4t$.

(d) $\frac{d^4w}{dt^4} - 8\frac{d^2w}{dt^2} + 16w = 0$.

If $\frac{d^4w}{dt^4} - 8\frac{d^2w}{dt^2} + 16w = 0$, then $w = C_1e^{2t} + C_2te^{2t} + C_3e^{-2t} + C_4te^{-2t}$.

(AB 2) Solve the following initial-value problems. Express your answers in terms of real functions.

(a) $9\frac{d^2v}{dt^2} + 6\frac{dv}{dt} + 2v = 0$, $v(0) = 3$, $v'(0) = 2$.

If $9\frac{d^2v}{dt^2}+6\frac{dv}{dt}+2v = 0$, $v(0) = 3$, $v'(0) = 2$, then $v = 3e^{-t/3}\cos(t/3)$ $9e^{-t/3}\sin(t/3)$.

(b) $\frac{d^2u}{dt^2} + 10\frac{du}{dt} + 25u = 0$, $u(0) = 1$, $u'(0) = 4$.

If $\frac{d^2u}{dt^2} + 10\frac{du}{dt} + 25u = 0$, $u(0) = 1$, $u'(0) = 4$, then $u = e^{-5t} + 9te^{-5t}$.

(c) $8\frac{d^2f}{dt^2} - 6\frac{df}{dt} + f = 0$, $f(0) = 3$, $f'(0) = 1$.

If $8\frac{d^2f}{dt^2} - 6\frac{df}{dt} + f = 0$, then $f = e^{t/2} + 2e^{t/4}$.

(AB 3) A spring is suspended vertically. When an object with mass 5 kg is attached, it stretches the spring to a new equilibrium 4 cm lower. The system moves in a medium which damps it with damping constant 16 newton · seconds/meter. The object is pulled down an additional 2 cm and is released with initial velocity 3 meters/second upwards.

Write the differential equation and initial conditions that describe the position of the object. You may use 9.8 meters/second^2 for the acceleration of gravity.

(Answer 3) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$5\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 1225x = 0, \qquad x(0) = -0.04, \quad x'(0) = 3.$$

(AB 4) A 2-kg object is attached to a spring with constant 80 N/m and to a viscous damper with damping constant β. The object is pulled down to 10cm below its equilibrium position and released with no initial velocity.

Write the differential equation and initial conditions that describe the position of the object.

If $\beta = 20$ N · s/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

If $\beta = 30$ N · s/m, is the system overdamped, underdamped, or critically damped? Do you expect to see decaying oscillations in the solutions?

(Answer 4) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$2\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + 80x = 0, \qquad x(0) = -0.1, \quad x'(0) = 0.$$

If $\beta = 20$ N $\cdot$ s/m, then the system is underdamped, and we do expect to see decaying oscillations.

If $\beta = 30$ N $\cdot$ s/m, then the system overdamped, and we do not expect to see decaying oscillations.

(AB 5) A 3-kg object is attached to a spring with constant k and to a viscous damper with damping constant 42 N·sec/m. The object is set in motion from its equilibrium position with initial velocity 5 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object.

Find the values of k for which the system is underdamped, overdamped, and critically damped. Be sure to include units for k.

(Answer 5) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$3\frac{d^2x}{dt^2} + 42\frac{dx}{dt} + kx = 0, \qquad x(0) = 0, \quad x'(0) = -5.$$

The system is critically damped if $k = 147$ newtons/meter. It is underdamped if $k > 147$ newtons/meter and overdamped if $0 < k < 147$ newtons/meter.

(AB 6) A 4-kg object is attached to a spring with constant 70 N/m and to a viscous damper with damping constant β. The object is pushed up to 5cm above its equilibrium position and released with initial velocity 3 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object. Then find the value of β for which the system is underdamped, overdamped, and critically damped. Be sure to include units for β.

(Answer 6) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$4\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + 70x = 0, \qquad x(0) = 0.05, \quad x'(0) = -3.$$

Critical damping occurs when $\beta = 4\sqrt{70}$ $\mathrm{N \cdot s/m}$. The system is underdamped if $0 < \beta < 4\sqrt{70}$ $\mathrm{N \cdot s/m}$ and is overdamped if $\beta > 4\sqrt{70}$ $\mathrm{N \cdot s/m}$.

(AB 7) An object of mass m is attached to a spring with constant 80 N/m and to a viscous damper with damping constant 20 N·s/m. The object is pulled down to 5cm below its equilibrium position and released with initial velocity 3 m/s downwards.

Write the differential equation and initial conditions that describe the position of the object. Then find the value of m for which the system is underdamped, overdamped, or critically damped. Be sure to include units for m.

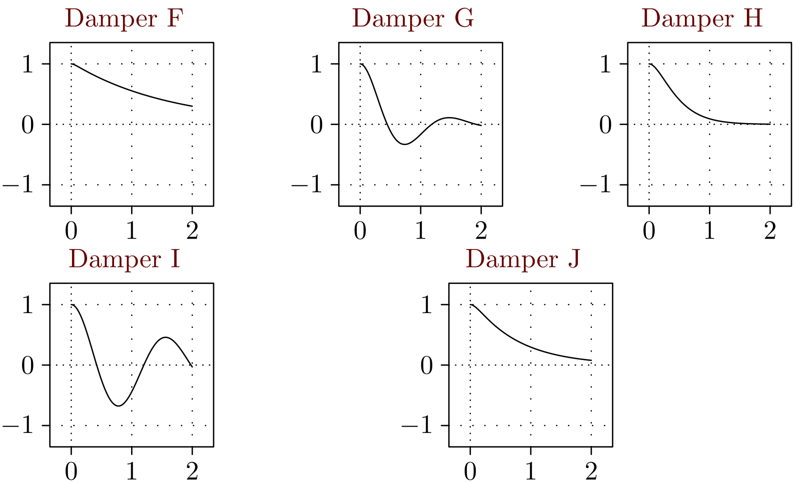
(Answer 7) Let t denote time (in seconds) and let x denote the object's displacement above equilibrium (in meters). Then

$$m\frac{d^2x}{dt^2} + 20\frac{dx}{dt} + 80x = 0, \qquad x(0) = -0.05, \quad x'(0) = -3.$$

Critical damping occurs when $m = \frac{5}{4}$ kg. The system is underdamped if $m > \frac{5}{4}$ kg and overdamped if $0 < m < \frac{5}{4}$ kg.

(AB 8) Five objects, each with mass 3 kg, are attached to five springs, each with constant 48 N/m. Five dampers with unknown constants are attached to the objects. In each case, the object is pulled down to a distance 1 cm below the equilibrium position and released from rest. You are given that the system is critically damped in exactly one of the five cases.

Here are the graphs of the objects' positions with respect to time:



(*a*) For which damper is the system critically damped?
The system is critically damped for Damper H.

(*b*) For which dampers is the system overdamped?
The system overdamped for Dampers J and F.

(*c*) For which dampers is the system underdamped?
The system underdamped for Dampers G and I.

(*d*) Which damper has the highest damping constant? Which damper has the lowest damping constant?
Damper F has the highest damping constant. Damper I has the lowest damping constant.

(AB 9) An object with mass 5 kg stretches a spring 4 cm. It is attached to a viscous damper with damping constant 16 newton · seconds/meter. The object is pulled down an additional 2 cm and is released with initial velocity 3 meters/second upwards.

Using only first derivatives, write the differential equations and initial conditions for the object's position. That is, write a first-order system involving the object's position. You may use 9.8 meters/second^2 for the acceleration of gravity.

(Answer 9) Let t denote time (in seconds), let x denote the object's displacement above equilibrium (in meters), and let v denote the object's velocity (in meters per second). Then

$$5\frac{dv}{dt} + 16v + 1225x = 0, \qquad \frac{dx}{dt} = v, \qquad x(0) = -0.04, \quad v(0) = 3.$$

(AB 10) Imperial stormtroopers and Rebel Alliance fighters battle each other on an open plain, where both groups can easily see and aim at all members of the other group. Every minute, each stormtrooper has a 2% chance of killing a rebel, and each rebel has a 5% chance of killing a stormtrooper. There are initially 4000 stormtroopers and 1000 rebels.

Write the initial value problem for the number of stormtroopers and rebels still alive.

(Answer 10) Let t denote time (in minutes), let S denote the number of stormtroopers, and let R denote the number of rebels.

Then

$$\frac{dR}{dt} = -0.02S, \quad \frac{dS}{dt} = -0.05R, \quad R(0) = 1000, \quad S(0) = 4000.$$

(AB 11) Jedi knights and Sith lords battle in a dense forest. Every minute, each Jedi has a 1% chance of finding each Sith lord. If a Jedi finds a Sith lord, they fight; the Jedi has a 60% chance of dying and the Sith lord has a 40% chance of dying. Initially there are 90 Jedi and 50 Sith lords.

Write the initial value problem for the number of Jedi and Sith lords still alive.

(Answer 11) Let t denote time (in minutes), let S denote the number of Sith lords, and let J denote the number of Jedi.

Then

$$\frac{dR}{dt} = -0.006JS, \quad \frac{dS}{dt} = -0.004JS, \quad J(0) = 90, \quad S(0) = 50.$$

(AB 12) A forest is inhabited by rabbits and foxes. Every year, each rabbit gives birth to 3 babies and each fox gives birth to 5 babies on average. Foxes try to catch and eat rabbits; each month, each fox has a 0.02% chance of catching each of the rabbits present in the forest. Each month, every fox that does not eat a rabbit starves to death.

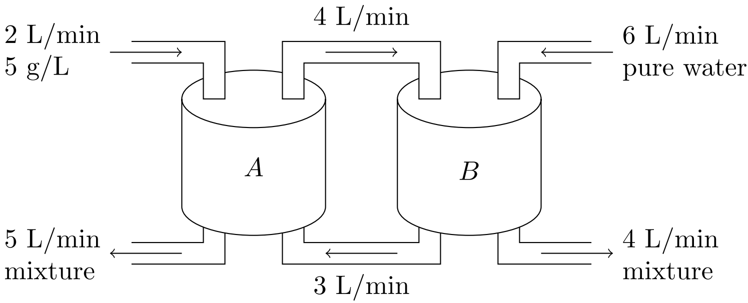
Write the differential equations for the number of foxes and rabbits in the forest.

(Answer 12) Let t denote time (in months), let R denote the number of rabbits, and let F denote the number of foxes.

Then

$$\frac{dR}{dt} = \frac{3}{12}R - 0.0002RF, \quad \frac{dF}{dt} = \frac{5}{12}F - (F - 0.0002RF).$$

(AB 13) Consider the following system of tanks. Tank A initially contains 200 L of water in which 3 kg of salt have been dissolved, and Tank B initially contains 300 L of water in which 2 kg of salt have been dissolved. Salt water flows into each tank at the rates shown, and the well-stirred solution flows between the two tanks and is drained away through the pipes shown at the indicated rates.



Write the differential equations and initial conditions that describe the amount of salt in each tank.

(Answer 13) Let t denote time (in minutes).

Let x denote the amount of salt (in grams) in tank A.

Let y denote the amount of salt (in grams) in tank B.

Then $x(0) = 3000$ and $y(0) = 2000$.

If $t < 50$, then

$$\frac{dx}{dt} = 10 - \frac{9x}{200-4t} + \frac{3y}{300+3t}, \qquad \frac{dy}{dt} = \frac{4x}{200-4t} - \frac{7y}{300+3t}.$$

(AB 14) Write the system

$$\frac{dx}{dt} = 6x + 8y, \qquad \frac{dy}{dt} = -2x - 2y$$

in matrix form.

(Answer 14)

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(AB 15) Write the system

$$\frac{dx}{dt} = 5y, \qquad \frac{dy}{dt} = 4y - x$$

in matrix form.

(Answer 15)

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

(AB 16) Find the solution to the initial-value problem

$$\frac{dx}{dt} = 6x + 8y, \qquad \frac{dy}{dt} = -2x - 2y, \qquad x(0) = -5, \quad y(0) = 3.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(Answer 16) If

$$\frac{dx}{dt} = 6x + 8y, \qquad \frac{dy}{dt} = -2x - 2y, \qquad x(0) = -5, \quad y(0) = 3$$

then

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{2t} \begin{pmatrix} 4t - 5 \\ -2t + 3 \end{pmatrix}.$$

(AB 17) Find the solution to the initial-value problem

$$\frac{dx}{dt} = 7x - \frac{9}{2}y, \qquad \frac{dy}{dt} = -18x - 17y, \qquad x(0) = 0, \quad y(0) = 1.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(Answer 17) If

$$\frac{dx}{dt} = 7x - \frac{9}{2}y, \qquad \frac{dy}{dt} = -18x - 17y, \qquad x(0) = 0, \quad y(0) = 1$$

then

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-20t} \begin{pmatrix} 3/20 \\ 9/10 \end{pmatrix} + e^{10t} \begin{pmatrix} -3/20 \\ 1/10 \end{pmatrix}.$$

(AB 18) Find the solution to the initial-value problem

$$\frac{dx}{dt} = -6x + 4y, \qquad \frac{dy}{dt} = -\frac{1}{4}x - 4y, \qquad x(0) = 1, \quad y(0) = 4.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(Answer 18) If

$$\frac{dx}{dt} = -6x + 4y, \qquad \frac{dy}{dt} = -\frac{1}{4}x - 4y, \qquad x(0) = 1, \quad y(0) = 4$$

then

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-5t} \begin{pmatrix} 15t + 1 \\ (15/4)t + 4 \end{pmatrix}.$$

(AB 19) Find the solution to the initial-value problem

$$\frac{dx}{dt} = -2x + 2y, \qquad \frac{dy}{dt} = 2x - 5y, \qquad x(0) = 1, \quad y(0) = 0.$$

Express your final answer in terms of real functions (no complex numbers or complex exponentials).

(Answer 19) If

$$\frac{dx}{dt} = -2x + 2y, \qquad \frac{dy}{dt} = 2x - 5y, \qquad x(0) = 1, \quad y(0) = 0$$

then

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-t} \begin{pmatrix} 4/5 \\ 2/5 \end{pmatrix} + e^{-6t} \begin{pmatrix} 1/5 \\ -2/5 \end{pmatrix}.$$

(AB 20) You are given that the general solution to the system

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 11 & -4 \\ 9 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

may be written as

$$\begin{pmatrix} x \\ y \end{pmatrix} = C_1 e^{5t} \begin{pmatrix} 2 \\ 3 \end{pmatrix} + C_2 e^{5t} \begin{pmatrix} 2t+1 \\ 3t+1 \end{pmatrix}.$$

Find the solution to the initial value problem

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 11 & -4 \\ 9 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$

(Answer 20) If

$$\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = \begin{pmatrix} 11 & -4 \\ 9 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix},$$

then

$$\begin{pmatrix} x \\ y \end{pmatrix} = 4e^{5t} \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 7e^{5t} \begin{pmatrix} 2t+1 \\ 3t+1 \end{pmatrix} = e^{5t} \begin{pmatrix} 1-14t \\ 5-21t \end{pmatrix}.$$