Properties of exponentials

On this section, use this table; do not use a calculator.

$2^2 = 4,$	$2^3 = 8,$	$2^4 = 16,$	$2^5 = 32,$	$2^6 = 64,$	$2^7 = 128,$
$3^2 = 9,$	$3^3 = 27,$	$3^4 = 81,$	$3^5 = 243,$	$3^6 = 729,$	$3^7 = 2187,$
$5^2 = 25,$	$5^3 = 125,$	$5^4 = 625,$	$5^5 = 3125,$	$5^6 = 15625,$	
$6^2 = 36,$	$6^3 = 216,$	$6^4 = 1296,$	$6^5 = 7776,$		
$7^2 = 49,$	$7^3 = 343,$	$7^4 = 2401,$	$7^5 = 16807.$		

1. $\dfrac{(7^3)^4}{7^7}$

2. $7^5 \, 7^{-3}$

3. $\left(\dfrac{7}{3}\right)^3$

4. $3^5 \, 2^5$

5. $625^{1/2}$

6. 7^{-4}

7. 10^0

8. $4^{\log_4 15}$

9. $\log_8 8^6$

10. $\log_6 7776$

11. $\log_7 \sqrt{343}$

12. $\log_5 (25 \cdot 125)$

13. $\log_6 \dfrac{216}{1296}$

14. $\log_3 (243^4)$

15. $\log_7 \dfrac{1}{2401}$

16. $\log_{10} 100$

17. $\dfrac{\ln 16}{\ln 2}$

18. $\log_8 128$

Calculus review

Evaluate the following derivatives.

19. $\dfrac{d}{dx}(7x^5)$

20. $\dfrac{d}{dx}(x^8 - 3x^4)$

21. $\dfrac{d}{dx}\left(\sqrt{x} + \dfrac{3}{x^5}\right)$

22. $\dfrac{d}{dx}(12e^{-4x})$

23. $\dfrac{d}{dx}(2^x)$

24. $\dfrac{d}{dx}(4e^{-x^2})$

25. $\dfrac{d}{dx}\ln x$

26. $\dfrac{d}{dx}\ln(8x)$

27. $\dfrac{d}{dx}\ln(x^2 + 1)$

28. $\dfrac{d}{dx}\ln(e^x + 1)$

29. $\dfrac{d}{dx}\sin(x)$

30. $\dfrac{d}{dx}(3\cos x)$

31. $\dfrac{d}{dx}\tan(5x)$

32. $\dfrac{d}{dx}\cot(2x - 3)$

33. $\dfrac{d}{dx}\sec(x^2)$

34. $\dfrac{d}{dx}\csc(e^x)$

35. $\dfrac{d}{dx}3\arcsin x$

36. $\dfrac{d}{dx}\arctan(x/3)$

37. $\dfrac{d}{dx}\operatorname{arcsec}(x + 3)$

38. $\dfrac{d}{dx}(x^2 e^{3x})$

39. $\dfrac{d}{dx}(x^4 \cos(2x))$

40. $\dfrac{d}{dx}\dfrac{\sin x}{x}$

41. Let $f(x) = x^7 g(x)$, where $g(1) = 5$ and $g'(1) = 3$. Find $f'(1)$.

42. Let $f(x) = 3e^{-2x}g(x)$, where $g(0) = 2$ and $g'(0) = 4$. Find $f'(0)$.

43. Suppose that $y = y(x)$ is defined by the relation $y^5 + y = x$. Use implicit differentiation to find $\dfrac{dy}{dx}$.

44. Suppose that $y = y(x)$ is defined by the relation $y + x^2 y^3 = x^4$. Use implicit differentiation to find $\dfrac{dy}{dx}$.

Evaluate the following integrals.

45. $\displaystyle\int_1^{32} \sqrt[5]{x}\, dx$

46. $\displaystyle\int 2x^3 + \dfrac{3}{x^5}\, dx$

47. $\displaystyle\int 3e^{7x}\, dx$

48. $\displaystyle\int_{-e}^{-1} \dfrac{9}{x}\, dx$

49. $\displaystyle\int 3\sin(5x)\, dx$

50. $\displaystyle\int 8\cos(2x)\, dx$

Use u-substitution to evaluate the following integrals.

51. $\displaystyle\int_0^{\pi} \cos^4 x \sin x\, dx$

52. $\displaystyle\int e^{\sin 3x} \cos 3x\, dx$

53. $\displaystyle\int \dfrac{e^{7x}}{e^{7x} + 5}\, dx$

54. $\displaystyle\int \tan x\, dx$

55. $\displaystyle\int \cot x\, dx$

56. $\displaystyle\int \sec x\, dx = \int \dfrac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$

57. $\displaystyle\int \csc x\, dx = \int \dfrac{\csc^2 x + \csc x \cot x}{\csc x + \cot x}\, dx$

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Use integration by parts to evaluate the following integrals.

58. $\int x \sin(3x)\, dx$

59. $\int x^2 \ln x\, dx$

60. $\int e^x \sin x\, dx$

61. $\int \cos^2 x\, dx$

Use trigonometric substitution to evaluate the following integrals.

62. $\int \dfrac{1}{4 + x^2}\, dx$

63. $\int \sqrt{9 - x^2}\, dx$

64. $\int \dfrac{1}{\sqrt{x^2 - 1}}\, dx$

65. $\int \dfrac{1}{(x+3)^2 + 16}\, dx$

Use partial fractions to evaluate the following integrals.

66. $\int \dfrac{1}{9 - x^2}\, dx$

67. $\int \dfrac{4x^2}{(x+1)(x^2+1)}\, dx$

68. $\int \dfrac{x + 5}{(x+1)(x+2)^2}\, dx$

1. $\dfrac{(7^3)^4}{7^7} = 16807.$

2. $7^5\,7^{-3} = 49.$

3. $\left(\dfrac{7}{3}\right)^3 = \dfrac{343}{27}.$

4. $3^5\,2^5 = 1296.$

5. $625^{1/2} = 25.$

6. $7^{-4} = \dfrac{1}{2401}.$

7. $10^0 = 1.$

8. $4^{\log_4 15} = 15.$

9. $\log_8 8^6 = 6.$

10. $\log_6 7776 = 5.$

11. $\log_7 \sqrt{343} = \dfrac{3}{2}.$

12. $\log_5(25 \cdot 125) = 5.$

13. $\log_6 \dfrac{216}{1296} = -1.$

14. $\log_3(243^4) = 20.$

15. $\log_7 \dfrac{1}{2401} = -4.$

16. $\log_{10} 100 = 2.$

17. $\dfrac{\ln 16}{\ln 2} = 4.$

18. $\log_8 128 = \dfrac{7}{3}.$

19. $\dfrac{d}{dx}(7x^5) = 35x^4.$

20. $\dfrac{d}{dx}(x^8 - 3x^4) = 8x^7 - 12x^3.$

21. $\dfrac{d}{dx}\left(\sqrt{x} + \dfrac{3}{x^5}\right) = \dfrac{1}{2\sqrt{x}} - \dfrac{15}{x^6}.$

22. $\dfrac{d}{dx}(12e^{-4x}) = -48e^{-4x}.$

23. $\dfrac{d}{dx}(2^x) = 2^x \ln 2.$

24. $\dfrac{d}{dx}(4e^{-x^2}) = -8xe^{-x^2}.$

25. $\dfrac{d}{dx}\ln x = \dfrac{1}{x}.$

26. $\dfrac{d}{dx}\ln(8x) = \dfrac{1}{x}.$

27. $\dfrac{d}{dx}\ln(x^2+1)=\dfrac{2x}{x^2+1}.$

28. $\dfrac{d}{dx}\ln(e^x+1)=\dfrac{e^x}{e^x+1}.$

29. $\dfrac{d}{dx}\sin(x)=\cos x.$

30. $\dfrac{d}{dx}(3\cos x)=-3\sin x.$

31. $\dfrac{d}{dx}\tan(5x)=5\sec^2(5x).$

32. $\dfrac{d}{dx}\cot(2x-3)=-2\csc^2(2x-3).$

33. $\dfrac{d}{dx}\sec(x^2)=2x\sec(x^2)\tan(x^2).$

34. $\dfrac{d}{dx}\csc(e^x)=-e^x\csc(e^x)\cot(e^x).$

35. $\dfrac{d}{dx}3\arcsin x=\dfrac{3}{\sqrt{1-x^2}}.$

36. $\dfrac{d}{dx}\arctan(x/3)=\dfrac{3}{9+x^2}.$

37. $\dfrac{d}{dx}\operatorname{arcsec}(x+3)=\dfrac{1}{|x+3|\sqrt{(x+3)^2-1}}.$

38. $\dfrac{d}{dx}(x^2 e^{3x})=2xe^{3x}+3x^2 e^{3x}.$

39. $\dfrac{d}{dx}(x^4\cos(2x))=4x^3\cos(2x)-2x^4\sin(2x).$

40. $\dfrac{d}{dx}\dfrac{\sin x}{x}=\dfrac{x\cos x-\sin x}{x^2}.$

41. $f'(x)=7x^6 g(x)+x^7 g'(x)$, so $f'(1)=7g(1)+g'(1)=38.$

42. $f'(x)=-6e^{-2x}g(x)+3e^{-2x}g'(x)$, so $f'(0)=-6g(0)+3g'(0)=0.$

43. If $y^5+y=x$, then $\dfrac{dy}{dx}=\dfrac{1}{5y^4+1}.$

44. If $y+x^2 y^3=x^4$, then $\dfrac{dy}{dx}=\dfrac{4x^3-2xy^3}{3x^2 y^2+1}.$

45. $\displaystyle\int_1^{32}\sqrt[5]{x}\,dx=\dfrac{5}{6}x^{6/5}\Big|_1^{32}=\dfrac{5}{6}32^{6/5}-\dfrac{5}{6}=\dfrac{315}{6}.$

46. $\displaystyle\int 2x^3+\dfrac{3}{x^5}\,dx=\dfrac{1}{2}x^4-\dfrac{3}{4x^4}+C.$

47. $\displaystyle\int 3e^{7x}\,dx=\dfrac{3}{7}e^{7x}+C.$

48. $\displaystyle\int_{-e}^{-1}\dfrac{9}{x}\,dx=9\ln|x|\Big|_{-e}^{-1}=9\ln 1-9\ln e=-9.$

49. $\displaystyle\int 3\sin(5x)\,dx=-\dfrac{3}{5}\cos(5x)+C.$

50. $\int 8\cos(2x)\,dx = 4\sin(2x) + C.$

51. $\int_0^\pi \cos^4 x \sin x \, dx = -\frac{1}{5}\cos^5 x \Big|_0^\pi = -\frac{1}{5}\cos^5 \pi + \frac{1}{5}\cos^5 0 = \frac{2}{5}.$

52. $\int e^{\sin 3x}\cos 3x\,dx = \frac{1}{3}e^{\sin 3x} + C.$

53. $\int \frac{e^{7x}}{e^{7x}+5}\,dx = \frac{1}{7}\ln(e^{7x}+5) + C.$

54. $\int \tan x \, dx = \int \frac{\sin x}{\cos x}\,dx.$ Using the substitution $u = \cos x$, we see that $\int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C = \ln|\sec x| + C.$

55. $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx.$ Using the substitution $u = \sin x$, we see that $\int \frac{\cos x}{\sin x}\,dx = \ln|\sin x| + C.$

56. $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

57. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C.$

58. $\int x\sin(3x)\,dx = -\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C.$

59. $\int x^2\ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C.$

60. $\int e^x\sin x\,dx = \frac{1}{2}e^x\sin x - \frac{1}{2}e^x\cos x + C.$

61. $\int \cos^2 x\,dx = \frac{1}{2}\sin x\cos x + \frac{1}{2}x + C.$

62. $\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan\frac{x}{2} + C.$

63. $\int \sqrt{9-x^2}\,dx = \frac{1}{2}x\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3} + C.$

64. $\int \frac{1}{\sqrt{x^2-1}}\,dx = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln\left(\sqrt{x^2-1} + x\right) + C.$

65. $\int \frac{1}{(x+3)^2+16}\,dx = \frac{1}{4}\arctan\frac{x+3}{4} + C.$

66. $\int \frac{1}{9-x^2}\,dx = \int \frac{1/6}{3-x} + \frac{1/6}{3+x}\,dx = -\frac{1}{6}\ln|3-x| + \frac{1}{6}\ln|3+x| + C.$

67. $\int \frac{4x^2}{(x+1)(x^2+1)}\,dx = \int \frac{2}{x+1} + \frac{2x-2}{x^2+1}\,dx = 2\ln|x+1| + \ln(x^2+1) - 2\arctan x + C.$

68. $\int \frac{x+5}{(x+1)(x+2)^2}\,dx = \int \frac{4}{x+1} - \frac{4}{x+2} - \frac{3}{(x+2)^2}\,dx = 4\ln|x+1| - 4\ln|x+2| + \frac{3}{x+2} + C.$